Abstract

VORA, POORVI L. Optimization Criteria and Numerical Analysis in the Design of Colour Scanning Filters (Under the direction of Prof. H. Joel Trussell).

The problem of the evaluation, design and analysis of colour scanning filters is addressed in this dissertation. The problem is posed within the framework of the vector space approach to colour systems. A data-independent measure of the goodness of a set of colour scanning filters is developed and tested. The measure may be used to evaluate any set of filters used to approximate a basis for a space of any dimension. The measure is shown to be a generalization of Neugebauer's q-factor. The data-independent measure is extended to a family of data-dependent measures which are related to the mean-square tristimulus error over a data set. Simulations indicate that both the data-independent measure and the data-dependent measures are good indicators of a perceptual error measure, the ΔE_{Lab} error.

The data-independent measure is used as an optimization criterion to design colour scanning filters. The filters are modelled in terms of known, smooth, non-negative functions. The best filters are then trimmed using the gradient of the mean square ΔE_{Lab} error or the data-independent measure to obtain filters with a lower value of perceptual error measures, or a higher value of the data-independent measure. The results obtained are excellent. The second differential of the mean square ΔE_{Lab} error or the data-independent measure provide a means of calculating the sensitivity of the mean square ΔE_{Lab} error or the data-independent measure respectively to filter fabrication errors. Tolerances on the allowable change in the mean square ΔE_{Lab} error or the data-independent measure are used to define bounds on the filter fabrication errors at all wavelengths and at single wavelengths. Simulations indicate the validity of the bounds.

For my parents, my brother and my grandmother,
whose love, encouragement and faith in my abilities
have made this possible.

Optimization Criteria and Numerical Analysis in the Design of Colour Scanning Filters

by
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CHAPTER 1

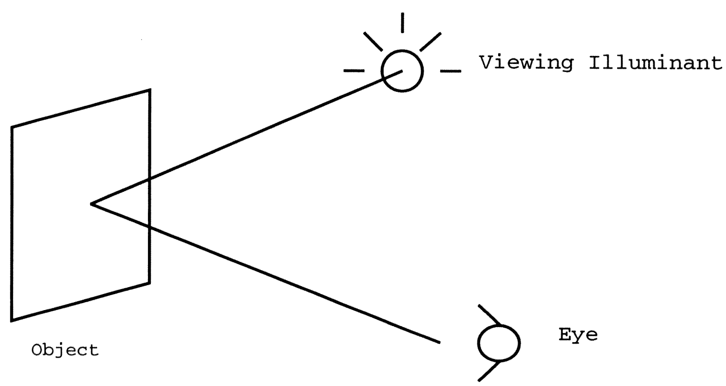
Introduction

Colour filters have several purposes including filtering for multiband image recording, illumination filtering for reproduction of specified viewing conditions and filtering for photographic special effects. Filters used for multiband image recording for the purpose of colour reproduction are referred to as colour scanning filters even though many modern imaging devices such as charge-coupled device (CCD) arrays do not 'scan'. These filters are used in colour reproduction systems like colour copy machines, colour printers, offset and gravure printing systems, etc. The purpose of colour measurements is to accurately record some physical properties of a signal so as to characterize its visual stimulus. This characterization is often used to reproduce a colour image on some display or printer. If the reproduction is viewed by a human observer, the measurements can be limited to those properties which allow the creation of an image which will appear the same as the original to the human eye.

The illuminant under which a colour reproduction is viewed is known as the viewing illuminant. It is well-known that three parameters are sufficient to characterize a colour visual stimulus under a particular viewing illuminant. These three parameters are generally dependent upon the viewing illuminant (much as the apparent colour of an object depends on the viewing illuminant). The three parameters, called tristimulus values, are defined by the CIE (Commission Internationale de l'Eclairage), as the inner product of the CIE matching functions and the spectrum of the object. Their measurement is one of the important applications of colour scanning. It is believed that the eye measures three parameters of a spectrum incident on the retina by obtaining the inner product of the object spectrum with the sensitivities of the three different kinds of cones in the eye. The CIE matching functions are not identical to the sensitivities of the human cones, but provide parameters that are equivalent to the measurements the eye makes. The CIE matching functions have been obtained experimentally, and are described in more detail in section 1.1.1. Measurements made with scanning filters which replicate the combined effect of the CIE matching functions and the viewing illuminant are known as CIE tristimulus values.

Fig. 1.1 illustrates the scanning process. The radiant power reflected off an object is focussed through a lens onto a beam-splitter. The beam-splitter consists of two half-silvered mirrors and two regular mirrors. The half-silvered mirrors reflect part of the light and transmit part of it, while the regular mirrors reflect all the light. The beam-splitter splits the radiation into three parts, each part directed to a scanning filter. The radiation passes through the scanning filter. Each scanning filter passes a fraction of the light incident on it. The fraction of the radiant power that is allowed through a filter is a function of the wavelength of the radiant power. The function itself varies from filter to filter. So, for example, the red filter will allow high fractions of incident radiant power of wavelengths in the 'red' region through, and will allow through only a small fraction of incident radiant power at other wavelengths. The output of a scanning filter goes to a photo-detector which records the total radiant power through the filter. The scanner thus provides three measurements, similar to the three measurements made by the three types of cones in the human eye.

Accurate scanning of a colour image can ensure that all relevant information about the colour stimulus of a signal is obtained. As the output of a colour reproduction



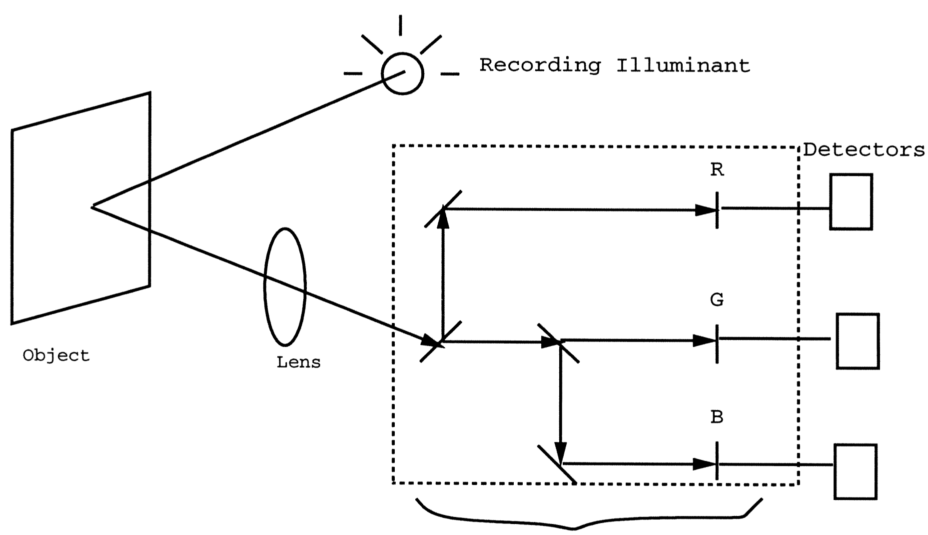


Figure 1.1: The Scanning Process

system can be only as accurate as the scanning filter measurements, the accuracy of a set of colour scanning filters determines an upper bound on the performance of the colour reproduction system. The set of scanning filters is hence a critical component of a colour reproduction system, and the design of an accurate set of colour scanning filters is a very important problem. While the CIE colour matching functions are well defined, it is not always possible to duplicate the functions or a linear combination of them with various filter materials. Figure 1.1 illustrates that the recording depends on the illuminant, the filters and the detector. This makes the fabrication of scanning filters more difficult. This dissertation addresses the problem of the design of a set of constructable colour scanning filters.

The general procedure for obtaining CIE tristimulus values of a reflective source is to obtain measurements from a set of three colour scanning filters. The reflective object is illuminated by the recording illuminant, the light reflected from the object passes through each of the scanning filters separately and three measurements of total radiant power allowed through each scanning filter are obtained from photodetectors in the path of the filtered light. These three measurements represent the combined effect of each filter, the recording illuminant, the light path and the object spectrum. A linear transformation is then performed on these three measurements to give estimates of the three CIE tristimulus values of the object under the viewing illuminant. This linear transformation is necessary because the scanning filters will, in general, not replicate the CIE matching functions exactly.

Colour scanning filters are most often designed to replicate the CIE matching functions. Any fabrication errors are compensated for by the above-mentioned linear transformation but this transformation is not incorporated into the design procedure. Fabricated filters are evaluated individually, in terms of how accurately each designed filter replicates the corresponding CIE matching function. It should be noted that the scanning filters need not always replicate the CIE matching functions. Scanning filter measurements need not be the actual CIE tristimulus values but may be an invertible linear transformation of the CIE tristimulus values so that a perfect set of scanning filters will provide data from which the CIE tristimulus values may be uniquely determined. Hence, the scanning filters need not be exact duplicates of the colour matching functions but need be only a nonsingular transformation of them. This fact is used in determining the filters used in television and other optical applications. Television filters are approximately the transformation of the CIE colour matching functions which would result from using the CRT phophors as the primary sources for the colour matching experiment described in section 1.1.1. MacAdam developed a transformation of the colour matching functions which resulted in a 'most orthogonal' set of filters as described in section 2.5. Besides this, there has been no other effort to evaluate a set of filters based on how close the set comes to being within a non-singular transformation of the CIE matching functions. This dissertation proposes a design procedure that incorporates the idea of the linear transformation into the design.

This dissertation addresses the problem of the evaluation, design and analysis of a set of colour scanning filters. Data-independent and data-dependent measures of the performance of a set of colour scanning filters are developed. The validity of these measures is demonstrated through simulations. The data-independent measure is used to define an optimization criterion for the design of colour scanning filters. A number of sets of smooth, physically realizable colour scanning filters are designed for two different actual scanner characteristics. These filters are trimmed to obtain smooth filters with low perceptual errors. A sensitivity analysis of the filter sets provides bounds on maximum possible fabrication errors given tolerances in the change in average perceptual error or in the value of the data-independent measure. These bounds are tested through simulations.

This chapter is organized as follows. Section 1.1 deals with colour preliminaries, including the definition of the CIE matching functions, and also the goals of colour scanning. Section 1.2 deals with the different types of optical filters. Section 1.3 elaborates on the relationship between the colour scanning problem and the problem of multi-band image recording in satellite imaging for remote sensing. Section 1.4 presents an outline of the dissertation.

1.1 Colour Preliminaries

Measurements obtained from a set of colour scanning filters should completely characterize the visual stimulus of the colour scanned. This implies that two different stimuli look alike under certain specified observing conditions if and only if the measurements from a perfect set of scanning filters are identical for both stimuli. Experimental work by colour scientists, psychologists and biologists [3, 42] indicates that three parameters are sufficient to define a colour stimulus completely for the human visual system, given a viewing illuminant. The three types of cones present in the human eye also indicate that three parameters are sufficient. One source of information about the responses to colour stimuli of the human visual system is the 'colour matching' experiment. Let us consider the basis for the requirements of colour scanning filters.

1.1.1 Colour Matching and the CIE Colour Matching Functions

In the colour matching experiment [42, pg. 118-143], fixed colour stimuli are sought to be matched with an additive mixture of three 'primary' colour stimuli, by a human observer. The human observer may vary the radiant power (defined as [42, pg. 690] radiant energy emitted, transferred, or received through a surface in unit time interval) of each of the three primary stimuli. A colour match is said to occur when the human observer cannot detect a difference between the two visual stimuli. The colour stimulus to be matched and the colour stimulus consisting of an additive mixture of three primary colour stimuli are presented as patches of contiguous light. The patches are similar in size and shape. The results of the experiment indicate that all colour stimuli may be matched in one of the following ways:

1. By adding all three primary stimuli
2. By adding one or two of the primary stimuli to the colour to be matched, and matching the result with an additive mixture of the remaining primary(ies).

The visual stimulus of any colour can be represented as a linear combination of the visual stimuli of the three primaries. The linear combination allows for a representation of the 'subtraction' of a primary stimulus, which is equivalent to the mixing of a primary stimulus with the colour to be matched, stated in the second method above.

Given the conditions of observation, the amount of radiant power of each of the primaries needed to match a particular colour defines the colour stimulus completely. As there are three primaries, there are three different values of radiant power. The three values of radiant power for a particular colour are known as the tristimulus values of the colour with respect to the primaries used. It should be noted that the amount of radiant power of each primary required to match a particular colour vary among observers and are dependent on observational conditions. Among the factors that would influence a colour match are the angle subtended by the matching field at the retina, the position of the image on the retina, and previous exposure of the eye to light. Light of very high brightness might affect distinct areas of the retina differently, leading to the appearance of a colour match, when such a match would not occur in the absence of previous exposure to bright light. Light of very high brightness also leads to 'saturation', where all colours appear white.

About 5% of all human observers are colour-defective [42, pg. 119]. Some of these require only two primary stimuli to match all colours. Such observers are called dichromats. Still others require only one primary stimulus to match all colours. Such observers are not able to discriminate among colours, and their matching is based on brightness. Such observers are called monochromats. While the colour sensitivities of humans differ, it is desirable to have a standard for measurements and quantitative colour description. The CIE has established a standard human observer named the CIE 1931 Standard Colorimetric Observer based on the colour matching experiments of Guild [42] and of Wright [42]. This standard was updated in 1964 based on the experiments of Stiles and Burch [42] and Speranskaya [42].

Monochromatic radiation is defined as radiation with radiant power concentrated in one wavelength value. The CIE primaries are defined as monochromatic stimuli R, G and B, at wavelengths 700 nm, 546.1 nm, and 435.8 nm respectively. The radiant powers of the monochromatic stimuli are in the ratios 72.1:1.4:1 respectively. Let the expression

$$f \Longleftrightarrow g$$

denote a colour match as apparent to the human eye. The experiments performed to determine the CIE Standard Observer consist of matching monochromatic stimuli at 5 nm intervals from 380nm to 760 nm using the primaries, R, G, B. Let a monochromatic stimulus at wavelength λ be denoted $e(\lambda)$. Let $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ be the amounts of primaries R, G and B required to match $e(\lambda)$. This may be expressed as:

$$\bar{r}(\lambda)R \;+\; \bar{g}(\lambda)G \;+\; \bar{b}(\lambda)B \Longleftrightarrow e(\lambda)$$

The functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, are functions of the wavelength λ of the monochromatic stimulus and form the CIE RGB colour matching functions. As the data is taken at 5 nm intervals, interpolation is necessary to give continuous functions of wavelength. These functions for the standard observer viewing a 2^0-bipartite visual field [42] are plotted in Figure 1.2 [42, pg. 124]. The wavelength plotted on the x-axis indicates the wavelength of the monochromatic stimulus, and hence, to some degree, its 'colour'. The tristimulus values on the y-axis are simply multiples of a signle unit of the respective primaries required to match the particular monochromatic stimulus. For example, about 1.8 units of primary R, 0.3 units of primary G and 0 units of primary B are required to match a monochromatic stimulus of one unit at wavelength 600 nm. The negative values of the function $\bar{r}(\lambda)$ indicate that, for particular values of the wavelength of the monochromatic stimulus, the primary R had to be shone on the object to make a colour match possible. In the linear model, this implies a negative value of the radiant power of the primary R in the linear combination of the three primaries.

It may be noted that the CIE observer has been defined after experiments with only 49 human observers, and is hence lacking as an adequate representation of a standard observer. It is, however, the only existing standard. Figs. 1.3-1.5 show different

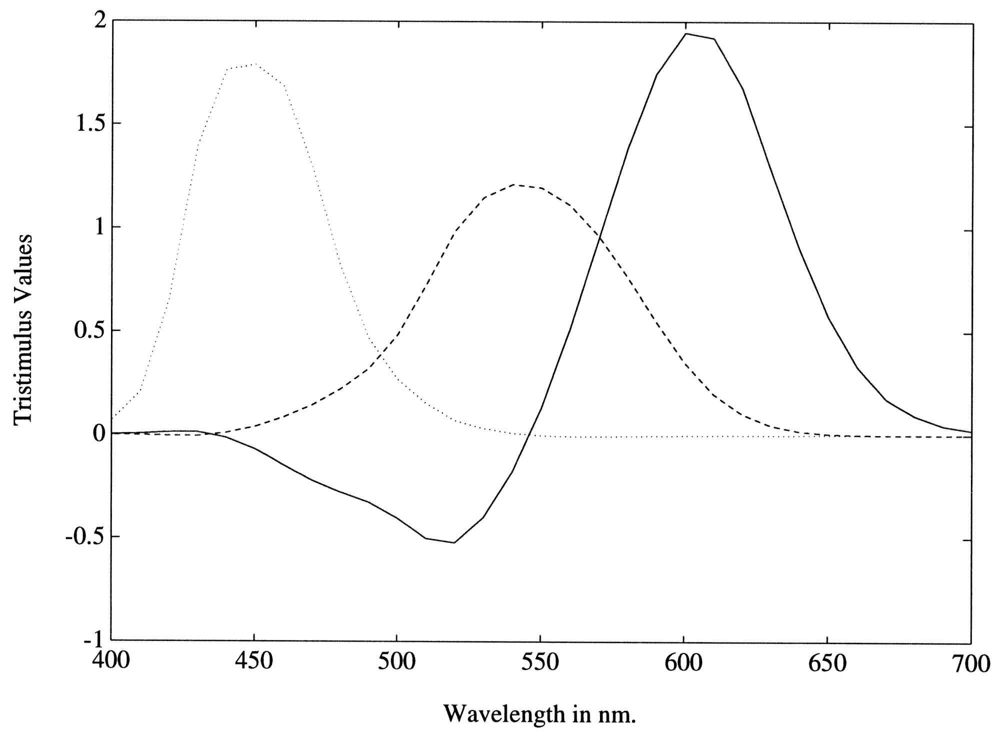


Figure 1.2: CIE RGB Matching Functions

matching functions for different individuals obtained by Fairchild [8], normalized to a maximum value of unity. The curves demonstrate that there is considerable variation among individual human systems with respect to the colour matching experiment.

Using the additive model for colour matching, and representing each colour as the sum of monochromatic stimuli of appropriate radiant power, it is possible to determine the tristimulus values of any radiant power spectrum with respect to the CIE primary stimuli R, G and B. Suppose $f(\lambda)$ is the radiant power spectrum of a visual stimulus. Its tristimulus values with respect to primaries R, G and B will be

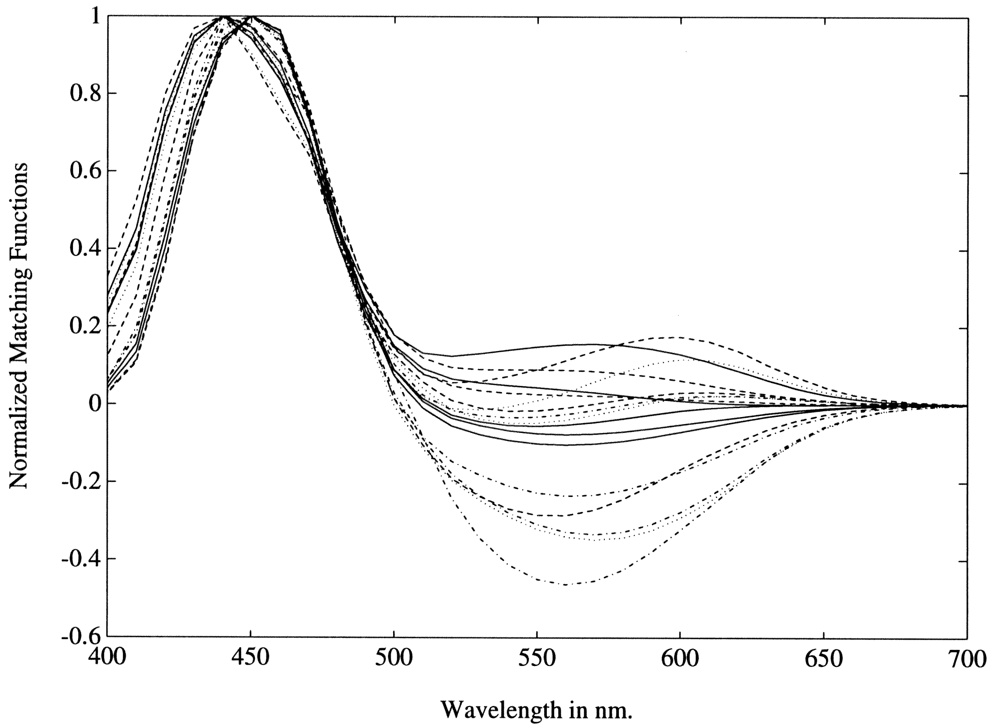


Figure 1.3: Experimental Data of Fairchild-1

[42, pg. 126]:

$$r = \int f(\lambda)\bar{r}(\lambda)d\lambda,$$

$$g = \int f(\lambda)\bar{g}(\lambda)d\lambda,$$

and,

$$b = \int f(\lambda)\bar{b}(\lambda)d\lambda.$$

This implies that r units of R, g units of G and b units of B are required to reproduce the effect of the radiant spectrum $f(\lambda)$ on the human visual system, or to produce a colour match with the colour represented by $f(\lambda)$. It is believed that the human

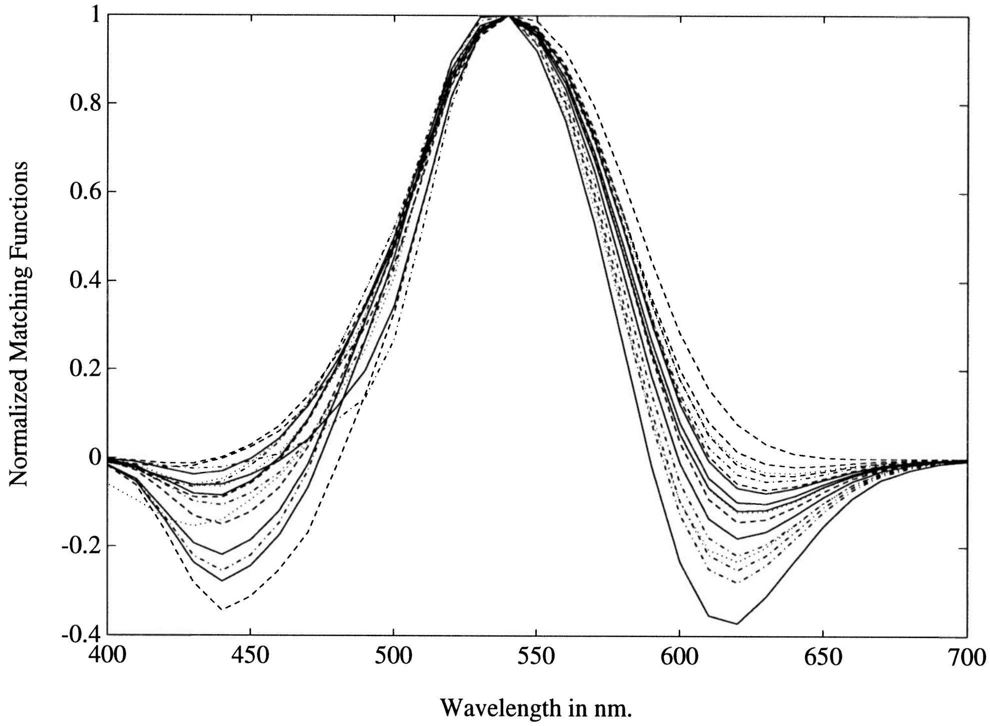


Figure 1.4: Experimental Data of Fairchild-2

visual system uses measurements similar to the tristimulus values mentioned above, to determine colour matches. It may be shown [31] that the tristimulus values defined above provide measurements equivalent to those obtained by the human visual system. Although the functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ do not represent the sensitivities of the human cones, they are within a linear transformation of the cone sensitivities.

For the set of matching functions, $\bar{m}_1(\lambda)$, $\bar{m}_2(\lambda)$, $\bar{m}_3(\lambda)$ a set of primaries may be defined as those spectra M_1, M_2 and M_3, such that

$$\int M_i(\lambda)\bar{m}_i(\lambda)d\lambda = 1 \quad i = 1,2,3 \tag{1.1}$$

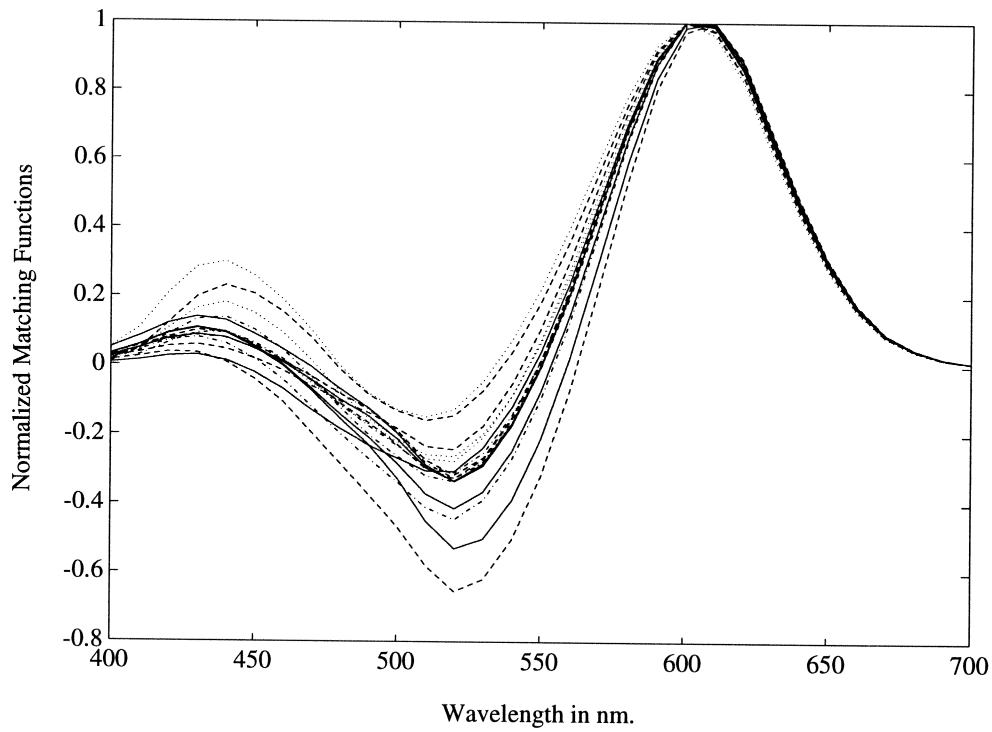


Figure 1.5: Experimental Data of Fairchild-3

and

$$\int M_i(\lambda)\bar{m}_j(\lambda)d\lambda \; = \; 0 \quad i \neq j \tag{1.2}$$

For a given set of matching functions, the set of primaries is not unique [31], and the primaries R, G, B are not the only primaries which would produce the matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$.

The transmissivity of an optical filter is defined as the ratio of emergent radiant flux to incident radiant flux, and is a function of wavelength. The output of a photodetector irradiated with light of spectral distribution $f(\lambda)$ passed through a filter with transmissivity $m(\lambda)$ is $\int f(\lambda)m(\lambda)d\lambda$, the total radiant power through the filter.

This implies that tristimulus values may be determined easily if filters with transmissivities replicating $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ may be manufactured. This, however, is not possible because $\bar{r}(\lambda)$ has negative values. To circumvent this problem, the CIE defined $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$, which are non-negative functions obtained through a non-singular linear transformation of $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$. The functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$, are tristimulus values of monochromatic stimuli of wavelength λ, with respect to another set of primaries, named X, Y, and Z. The spectrum of a primary represents the radiant power as a function of wavelength, and must hence be non-negative. It is not possible to obtain non-negative solutions $M_i(\lambda)$ to the equations (1.1) and (1.2) if the m_i represent the functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$. The primaries X, Y, Z are hence not realizable. As the functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ are non-negative, they may be realized, in theory, by transmissive filters. These functions are called the CIE XYZ matching functions, or simply the CIE matching functions, and are plotted in Fig. 1.6 [42, pg. 137].

The CIE tristimulus values of a radiant power spectrum $f(\lambda)$ are:

$$x = \int f(\lambda)\bar{x}(\lambda)d\lambda, \qquad (1.3)$$

$$y = \int f(\lambda)\bar{y}(\lambda)d\lambda,$$

and,

$$z = \int f(\lambda)\bar{z}(\lambda)d\lambda.$$

As the CIE matching functions are within a non-singular linear transformation of $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$, two colour stimuli will have identical values of r, g, b if and only if they have identical values of x, y, z. Similarly, it is shown in [31] that two colour stimuli will have identical values of x, y, z if and only if the outputs of the three

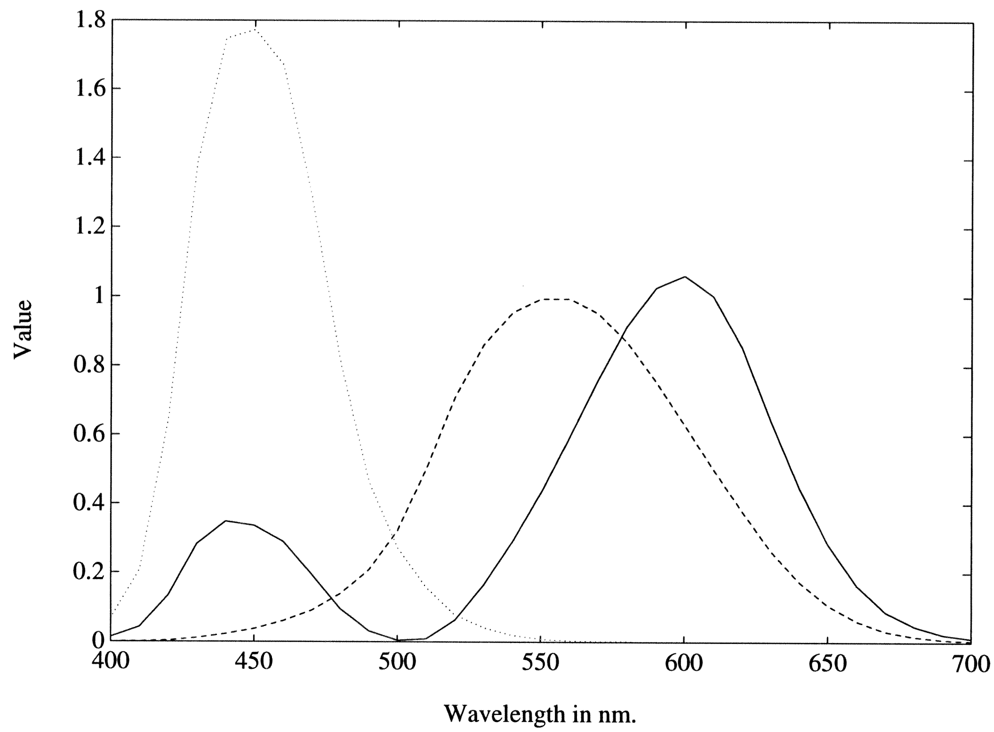


Figure 1.6: CIE XYZ Matching Functions

types of cones in the human eye are identical for the two stimuli. It can also be shown [31] that the cone sensitivities are within a non-singular transformation of the CIE matching functions.

The CIE defined a standard based on the convenience of manufacturing a set of colour scanning filters. There is no reason why all colour scanning filter sets should replicate the CIE matching functions. A linear transformation of the CIE matching functions could result in functions that are easier to manufacture than the CIE matching functions themselves. Further, the dependence on primaries implies that the CIE tristimulus values might not be the requirement of all scanning systems.

For example, colour consoles may be modelled as additive colour displays (see section 2.3). The primaries for colour consoles cannot be the primaries X, Y, Z because these primaries are not realizable. If a colour scanner is to be used with a colour console as the display device, the scanner measurements should determine the tristimulus values with respect to the primaries defined by the phosphors in the console. In general, these tristimulus values will be distinct from the CIE tristimulus values. These console-specific tristimulus values may be determined either directly by the set of scanning filters if the matching functions defined by the primaries are easily realizable, or by a linear transformation of measurements from an appropriate set of scanning filters.

The above discussion implies that the three parameters required to define the visual stimulus of a colour signal under a particular viewing illuminant are not necessarily just the CIE tristimulus values. This implies that the problem of the design of scanning filters is not required to be stated as the problem of the replication of the CIE colour matching functions.

1.1.2 Colour Spaces

A visual stimulus may be completely represented by three parameters for a particular viewing illuminant. The three tristimulus values, given a set of primaries and viewing conditions, represent the three coordinates of a point in a three-dimensional space. Such a three-dimensional space is known as tristimulus space. The perceptual error, which is the subjective difference between two colour stimuli as seen by the human eye, is not directly related to the euclidean distance between the two points in any tristimulus space. This has motivated the definition of a few 'uniform' colour spaces, where the perceptual error may be approximated by the euclidean distance between the points. The definitions of the uniform colour spaces have an experimental origin. The uniform colour space used in this dissertation is the CIE $L^*a^*b^*$ space [42, pg. 167]. Other uniform colour spaces are discussed in detail in [42, pg. 164], [25, pg. 70-71, 85-88], [14, pg. 114-122]. The CIE $L^*a^*b^*$ space is defined as the space of points obtained by the following transformations of the CIE tristimulus values, x, y, z defined in equation (1.3).

$$\mathcal{F}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} = \Upsilon \begin{bmatrix} (\frac{x}{x_n})^{1/3} \\ (\frac{y}{y_n})^{1/3} \\ (\frac{z}{z_n})^{1/3} \end{bmatrix} - \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix} \tag{1.4}$$

where

$$\Upsilon = \begin{bmatrix} 0 & 116 & 0 \\ 500 & -500 & 0 \\ 0 & 200 & -200 \end{bmatrix} \tag{1.5}$$

and x_n, y_n and z_n are normalization factors, the tristimulus values of a nominal 'white point'. The 'white point' represents the tristimulus values of a standard illuminant, or a reference background point. The above formula is used when $\frac{x}{x_n}, \frac{y}{y_n}, \frac{z}{z_n} \geq 0.01$. When $\frac{x}{x_n}, \frac{y}{y_n}, \frac{z}{z_n} < 0.01$,

$$\mathcal{F}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} = \mho \begin{bmatrix} f(\frac{x}{x_n}) \\ f(\frac{y}{y_n}) \\ f(\frac{z}{z_n}) \end{bmatrix} + \begin{bmatrix} 903.3\frac{y}{y_n} \\ 0 \\ 0 \end{bmatrix} \tag{1.6}$$

where

$$\mho = \begin{bmatrix} 0 & 0 & 0 \\ 500 & -500 & 0 \\ 0 & 200 & -200 \end{bmatrix} \tag{1.7}$$

and

$$f(t) = \begin{cases} (t)^{\frac{1}{3}} & t > 0.008856 \\ 7.787(t) + \frac{16}{116} & t \leq 0.008856 \end{cases}$$

The perceptual error between two points is defined as the euclidean distance between two points in CIE $L^*a^*b^*$ space, and is called the ΔE_{Lab} value of the difference. A rule of thumb is that a ΔE_{Lab} error of 3 is perceptible, but smaller errors are not.

1.2 Optical Filters

Most of the information in this section follows the presentation in [42, pg. 30-48]. An optical filter is a device which 'changes selectively or nonselectively the spectral distribution of the incident radiant flux' [42]. The transmissivity of a combination of filters separated by air may be approximated by the product of the individual transmissivities. Two types of optical filters will be discussed in this dissertation, absorption filters and interference filters.

1.2.1 Absorption Filters

Absorption filters are most often made of glass, gelatin or liquids in which coloring agents are dissolved or suspended. The radiant flux is incident on a surface of the absorption filter, some flux is reflected at the surface of incidence, some of the flux is absorbed by the filter medium, some is reflected off the other surface, and the rest passes through. The transmissivity of an absorption filter depends on the angle of incidence of the radiant flux, polarization, the refractive index of the medium, the thickness of the medium, the temporal and spatial coherence of the radiant energy, the temperature and wavelength. An approximate formula for the transmissivity does not involve all these parameters. If $n(\lambda)$ is the ratio of the refractive indices of the filter and the surrounding medium, $m(\lambda)$ is the spectral absorptivity of the medium, d is the thickness of the medium, and

$$\rho(\lambda) = (\frac{n(\lambda) - 1}{n(\lambda) + 1})^2,$$

then the transmissivity of a homogenous, isotropic absorption filter is approximately:

$$T(\lambda) = (1 - \rho(\lambda))^2 10^{-dm(\lambda)}$$

If $n(\lambda) \sim 1$, then

$$T(\lambda) = r(\lambda)^d \tag{1.8}$$

where

$$r(\lambda) = 10^{-m(\lambda)}$$

Among the most widely used commercially available absorption filters are those made of coloured glass. The spectral transmissivities are documented in the manufacturer's catalogues, and characterize the filter for a particular thickness. Individual pieces often deviate from the predicted spectral transmissivities, and individual calibrations should be performed in instances where precision is important. The transmissivities are functions of temperature, although the change is reversible. Glass filters may fluoresce, especially when irradiated with ultraviolet radiant energy. This effect may be reduced by the selection of another filter which absorbs either the exciting radiant energy or the fluorescence. Glasses unprotected by thin-film coatings may tarnish on prolonged exposure to the atmosphere, high humidity, or high temperature. However, the spectral characteristics of these glasses are very stable if the glasses are protected.

Gelatin filters are made by mixing organic dyes in gelatin, coating the mixture on glass plates and removing the films off the plates when the mixture is dry. These filters are less expensive than glass filters and are easier to obtain in a given shape and size. They are less stable than glass filters, though, and are rarely used in precision colorimetry. Eastman Kodak manufactures the Kodak Wratten gelatin filters which may be obtained in a specified density. The density of a filter offers a variable that affects the spectral transmissivity(see equation (1.8)).

1.2.2 Interference Filters

Multiple reflections of a beam of radiant energy occur when the beam passes through an assembly of layers of different materials. These reflections interfere if the layers are thin enough. This interference forms the physical basis for the spectral transmissivities of interference filters. The general structure of an interference filter consists of layers of a reflecting surface (thin-film dielectric or metal) separated by dielectric spacer layers. Most interference filters are bandpass filters, in that a band of wavelengths is selectively transmitted, while radiant power at other wavelengths is absorbed. Unlike glass and gelatin filters, narrowband interference filters can be constructed for bandwidths as small as a few nanometers. While the filter transmissivities need to be fairly smooth, interference filters may be built to be bimodal, and are not necessarily unimodal. For examples of bimodal interference filters, see the filters that are manufacturable by Barr Associates and Eastman Kodak, in Chapter 4.

The spectral transmissivity curves of a bandpass interference filter are periodic as a function of wavelength. Hence, these curves consist of regularly separated passbands each corresponding to an order of interference of the filter. Bandpass interference filters are hence used with blocking filters which block the sidebands. Blocking filters are either absorption filters or other interference filters, and blocking serves to decrease the maximum transmissivity possible. This is one disadvantage of interference filters.

Because the filter effect depends upon the distance light travels in the layers, an increase in the angle of incidence leads to a decrease in the wavelength at which the spectral transmissivity is maximum. This property may cause serious calibration problems, and it is advisable to calibrate an interference filter in place. Interference filters can be designed to almost arbitrary functional forms and it is easier to use computer designs for their manufacture. The simulations by Barr Associates and Eastman Kodak presented in Chapter 4 illustrate this.

The fact that all scanning filters should be physically realizable, either as a combination of other filters (see section 2.6), or as constructable filters, implies that there are limitations on the transmissivity curves of the designed scanning filters. This implies that the definition of the filter design problem should include the restriction of physical realizability. This problem is dealt with in greater detail in Chapter 4.

1.3 Relation to Other Multi-Band Image Recording Problems

The problem of satellite imaging for remote sensing can be formulated as a problem similar to the problem of colour scanning. Remote sensing involves the recording of radiation reflected off the earth's surface, or emitted from it. There are two types of remote sensing, active and passive. Passive remote sensing involves the detection of energy that comes from a natural source, for example the earth itself or the sun. When the source of energy is the sun, the radiation detected by the sensor is radiation reflected off the earth. Active remote sensing involves the detection of energy that is directed towards the earth for the specific purpose of remote sensing. The radiation is reflected off the earth and is detected by sensors positioned to receive the radiation

[2].

The recording of sensed radiation is most often multispectral, i.e. there are many sensors with narrow-band filters that record the value [2]:

$$t_i = \int_{-\infty}^{\infty} g(\lambda) s_i(\lambda) d\lambda \tag{1.9}$$

where λ represents the wavelength of the radiation, g represents the radiation power sensed, and s_i represents the response of the i^{th} sensor with the corresponding narrow-band filter. Usually, $s_i(\lambda)$ is zero outside a small range of wavelength. The number of values of i is the number of bands in the recording, and is often 5 or larger, and may be as large as 200. The values t_i are used to characterize the nature of the object off which the radiation is reflected, and are sometimes said to form the spectral signature of the object. The function $g(\lambda)$ represents the reflected radiation $f(\lambda)$ after atmospheric degradation.

Note the resemblance of equation (1.9) to equation (1.3). The identification of certain kinds of surface properties of the earth (vegetation, soil, etc.) is closely linked to identifying the values t_i, much as the identification of the colour of an object is closely linked to its tristimulus values. The remote sensing problem may be interpreted as one of determining the 'spectral signature' of a particular surface area of the earth. Just as in the colour scanning problem, it is enough to determine a linear combination of the different t_i, and this is expected to lead to relaxed conditions for the optical filters used in the sensors manufactured for remote sensing purposes.

The results of the research presented in this dissertation are tested on simulations of colour scanning experiments, but they may easily be used for the design of filters for remote sensing applications. The analysis is presented in a general format which is not specific to colour scanning.

1.4 Dissertation Outline

To establish notation, an overview of the vector space approach to colour reproduction is presented in Chapter 2. A survey of existing methods for the design and evaluation of a set of colour scanning filters is also presented in Chapter 2.

The first requirement in the design procedure is a means of evaluating a set of proposed colour scanning filters. A data-independent measure that evaluates how well a set of filters allows accurate measurement of colour is proposed in Chapter 3. The reproduction error of a colour is defined, then an analysis of a measure of this error leads to the definition of a data-independent measure of the set of scanning filters. The basis of this measure is the relation between the vector space defined by the human visual system determined from the CIE colour matching functions and the vector space defined by the scanning filters. The measure is different from evaluation criteria used by other researchers [5, 6, 9, 41], in that it measures the set of filters as a whole and not the individual filters. The measure may be used to evaluate sets of more than three scanning filters and it may be used in other colour correction applications as well. This measure is extended to include dependence on the data set if it is fixed and known, as is often the case in practical colour scanning applications. Simulation results are presented to demonstrate the appropriateness of the measure.

The design of a set of three or more colour scanning filters is formulated as a parametrized optimization problem in Chapter 4. The optimization criterion is the measure of goodness developed in Chapter 3. Constraints are incorporated to ensure that the designed filter may be fabricated by existing fabrication methods. The design method can be used for any scanning system for which the colorimetric responses can be defined. The total system, including the lamps, light path and sensor characteristics, is taken into account. Simulations and results from actual hardware demonstrate the utility of the method.

Error analysis is performed to relate fabrication errors to resulting perceptual errors in the reproduced colours in Chapter 5. The analysis provides a basis for upper bounds on allowable fabrication errors given tolerances for perceptual errors or changes in the measure defined in Chapter 3. Upper bounds for fabrication errors are provided as a function of wavelength, providing the manufacturer with information on the required accuracy of fabrication as a function of wavelength.

A summary of the work done and conclusions are presented in Chapter 6. Suggestions for future work are also presented in this chapter.