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The Privacy Cost of the Second-Chance Offer

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ABSTRACT

This paper examines a generalization of a two-stage game common on eBay: an ascending-price auction followed by price discrimination (the *second chance offer*). High bids in the auction lead to high price offers during price discrimination, and a financial disadvantage in the second stage. The disadvantage depends on (a) the amount of information revealed to the seller in the first stage, and hence the extent of privacy protection provided and (b) whether the bidder is non-strategic (ignores the possibility of price discrimination) or rational. A *privacy cost* of one mechanism over another is defined and studied.

For the non-strategic bidder, the second chance offer provides a zero payoff. Addition of privacy protection (anonymity and bid secrecy) decreases revenue and increases expected payoff, with higher bidders benefiting more. Privacy protection can, however, decrease an individual bidder's payoff by shielding potential buyers from the seller and thus causing an opportunity loss.

If the bidder is rational, price discrimination results in a lower revenue than consecutive auctions, and is a bad strategy for the seller. Additionally, rational behavior provides more advantage to the bidder than does anonymity protection.

Categories and Subject Descriptors

K.4.4 [**Computers and Society**]: Electronic Commerce—*privacy*

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1. INTRODUCTION

This paper approaches the privacy problem as one of information revelation in multi-stage games. An example game is a series of electronic commerce transactions, such as auctions or fixed-price sales. The optimal play in a particular stage typically reveals information about future possible plays, and the revealed information may prove to be a disadvantage or advantage to the player. For example, information revelation can be used to advantage by the player to create a reputation as a serious buyer of high quality goods, or to obtain accurate recommendations for interesting sale items. On the other hand, a bid in an auction may classify a losing bidder as willing to pay a very high price for the item. This information may be used at a later stage to charge the bidder a higher price than others for that or a similar item. In a loose sense, the types of stages that are likely to follow the current one determine whether the overall impact of information revelation is positive or negative, and an intelligent rational agent would use information about following stages while devising its strategy.

Informally speaking, the *privacy cost* of playing in stage X of a multi-stage game is the later economic disadvantage, if any, of revealing information in stage X. The disadvantage can be reduced by revealing less information through the use of cryptographic techniques – which, for example, may protect the identities of players. The disadvantage may also be reduced by making available automated rational agent tools because privacy costs are typically higher for *non-strategic players* – those who failed to consider the possibility of consequent stages while playing – than they are for *rational players* – those who optimize their strategy in each stage taking into consideration subsequent stages.

At this time, it is not clear whether customers value their personal information highly or take into consideration the future economic costs of revealing too much information in a single transaction. Experimental evidence cited in [2] describes how even those customers who profess to value their privacy do not assert its value in electronic interactions when doing so would inconvenience them or cost money. On the other hand, it appears that when the decisions regarding information revelation are simple and not to be made on a continual basis, customers *do* choose to assert their value for privacy. For example, many customers trade their grocery shopping profile for a small discount [15], while others do not. For another example, some individuals are willing to pay to be kept off telephone directories. It is possible that the extensive and continual nature of electronic data collection today makes it impossible, even for the most privacyconscious individual, to make the innumerable rational decisions required, manually and on a case-by-case basis, about whom to reveal information to and how much to reveal. If this is so, customers would behave differently in electronic interactions if automated tools, such as privacy agents, existed that could participate in transactions on their behalf. One reason such tools do not exist is that optimal strategies in games do not typically take into account the privacy cost of participating in the game, and even a theoretical examination of the effect of privacy costs on the outcomes of simple games does not exist.

This paper examines privacy costs in a simple two-stage game that is quite common on eBay: following the sale of an item in a normal ascending auction, a seller can, at no extra charge, make a *second chance offer* to a losing bidder, at a price equal to her highest (failed) bid. Second-chance offers are instances of price-discrimination[1] considered by some [13] to be the motivation for privacy infringement. When bidders are non-strategic and reveal their valuations[2] through highest failed bids in the first stage, the prices charged in the second provide the seller with his highest possible revenue. In such cases, the second-stage payoff[3] to the non-strategic bidder is at its minimum value of zero.

Frequent access to auctions is a recent phenomenon for the general population. Hence, it is very likely that a number of bidders playing the eBay game are not familiar with multi-stage games and behave largely non-strategically. (See, for example, [14], which describes how experimental evidence suggests that bidders do not behave strategically in two-stage auctions). This paper studies the strategies of both rational and non-strategic bidders, and examines the corresponding impacts on the bidder's payoff and the seller's revenue. In particular, it provides a definition of (comparative) privacy cost and examines the privacy cost to the bidder of being non-strategic over being rational. This provides a simple quantitative expression of the advantage, to the bidder, of using rational privacy agents in this game. It also provides insight into whether the attempt to provide privacy in online commerce ought to include both cryptographic and agent-centered approaches.

The eBay game provides minimal privacy to the bidder because it reveals the eBay identities and corresponding bids to the seller, and also allows the seller to contact individual losing bidder(s) with the second-chance offer. While the bidder may minimize her economic disadvantage through the use of rational agents and an optimal strategy, the economic disadvantage may also be reduced through external privacy protection. This paper enhances the eBay game with various types of privacy protection – such as anonymity and bid secrecy. It examines the impact of the privacy protection on the non-strategic bidder, and compares the impact with that of the use of rational agents. The results are illustrated using real bids from an eBay auction.

1.1 The Model

The model is that of a simple two-stage game – an auction followed by price discrimination – in which many bidders and a single seller each seek to maximize a utility function. Each bidder maximizes her expected *payoff*, and each seller maximizes his revenue. As is typical in auction theory, each bidder is characterized by her valuation, which is the most she is willing to pay. The seller attempts to determine valuations in the first stage so as to be able to charge the highest possible prices in the second stage. The model considers N bidders with *private valuations* $x \in [0, \omega]$, independent and identically distributed according to the uniform probability distribution function. This paper considers the two-stage game:

Stage I: An ascending bid first-price (English) *auction* for a single item.

Stage II: A *price discrimination* stage that occurs with probability $\alpha = 1$; k more copies of the same item are offered to k of the $N-1$ losing bidders at prices P_i, $i = 2, 3, ..k+1$. We assume that the number of extra items, k, is small, i.e. $k < \frac{N}{2}$. The paper does not consider a general probability of price discrimination α, however, the case $\alpha = 0$ is considered for comparison purposes, see Case Ω.

The bidders and the seller all know that the valuations are uniformly distributed over $[0, \omega]$. The bidders do not know any valuations other than their own. The seller knows no valuations.

The paper considers the following models for the bidder strategies.

1. *Non-strategic Bidders:* All bidders treat the auction as being independent of Stage II, and bid in Stage I as rational bidders unaware of Stage II.

2. *Rational Bidders:* All bidders bid an optimal strategy for the two-stage game, taking the seller's optimal strategy into consideration.

In both cases, the seller is assumed to know the strategy of the bidders, and devises an optimal strategy to determine P_i.

The paper compares the above two-stage game to a game of $k+1$ stages – consecutive, independent auctions for the $k+1$ items – where neither the bidders nor the seller use information across stages.

Game Ω *No Price Discrimination; Consecutive Independent Auctions:* This $k = 1$-stage game is the hypothetical base line game where the probability of price discrimination is $\alpha = 0$, the seller holds $k+1$ consecutive independent auctions, and the bidder, when bidding in a single auction, does not use knowledge of the previous or the subsequent ones. It can occur, for example, when the bidder repeatedly tries auctions for the same object till she wins it, never knows what it sold for, has a private valuation, and does not know that she is always interacting with the same group of bidders across many auctions. While it is hypothetical, and perhaps unrealistic, its purpose is to provide a base line with which to compare the impact of information on strategies, revenues and payoffs.

Further, this paper compares the following cases of the two-stage eBay game among one another and with Game Ω.

Case A. *Price Discrimination; Non-strategic Bidders:* The bidder is non-strategic. Bids and corresponding bidder pseudonyms are known to all bidders and to the seller. The bidder pseudonyms are distinct from contact identities, and are made available in order to link bids from the same bidder. Hence bidders and seller may tell when a particular bidder drops out. Only the seller has the corresponding

[1]Price discrimination is the practice of charging different prices to different customers based on estimates of their willingness to pay for the good

[2]A bidder's valuation of an item is the highest she is willing to pay for the item

[3]A bidder's payoff is the difference between the highest she is willing to pay and the price she actually pays

contact identities and may contact each bidder individually. Nothing else is known to anyone.

This is similar to the *private auction* option of eBay[4], and is the base price discrimination case to which all other price discrimination cases (Cases B-D) are compared.

Case B. *Price Discrimination; Rational Bidders:* Case A, except that the bidder is rational. This case is examined to determine the advantage of rationality to the bidder.

Case C. *Price Discrimination; Non-strategic Bidders and Anonymity:* Case A, except that the seller can contact the bidders only as a group, and exactly once after the auction. Thus, as in Case A, each losing bidder may be approached at most once. Unlike Case A, the losing bidders are indistinguishable from one another and hence anonymous. This hypothetical case is examined to determine the advantage of anonymity to the non-strategic bidder.

Case D. *Price Discrimination; Non-strategic Bidders, Anonymity and Bid Secrecy:* Case C, except that the seller does not know the value of individual bids, and knows only the final selling price. In addition to the losing bidders being indistinguishable, the group of losing bidders is also indistinguishable from other groups with the same winning bid. This case is examined to determine the advantage of anonymity *and* bid secrecy to the non-strategic bidder.

The paper does not consider a *public* model – where bids and corresponding bidder identities are known not only to the seller, but also to the public – because it is inconsistent with a two-stage game. In a public model, anyone can contact the bidders individually, and subsequent interactions with *anyone* would be affected by the bids in this auction. The modeling of the cost of this information revelation is beyond the scope of this paper (though the authors believe this paper provides a first step towards approaching the problem). The public model is similar to the regular eBay auction, where any seller with an eBay account may contact bidders using their eBay identities. The main difference between the public and private eBay auctions, or a public auction and Case A, is that the price discrimination stage is a monopoly in Case A and in the private eBay auction.

The paper also does not consider the case when the seller does not price discriminate (i.e. holds $k + 1$ consecutive auctions) but the bidder uses information across stages (i.e. the auctions are not independent). This would correspond to the case of bidders using information when the seller does not. It is not a good model of the eBay game, and does not provide as much insight into the privacy problem as do the cases considered in this paper.

A recent unpublished working paper [14] proves the non-existence of pure equilibria in Case B of this game when two rational bidders bid for two items. It also shows that the seller is better off not price-discriminating when the bidders are rational. Similarly, [3] also describes how price discrimination is a suboptimal strategy for the seller. While the results of our paper are consistent with these, our paper goes further by examining the situation of the non-strategic bidder, privacy protection, the case of many bidders and many copies of the same item. Further, our paper provides a quantitative economic definition of privacy, and quantitative comparisons of various types of privacy protection.

[4]The eBay private auction provides somewhat less information to the bidder because it does not provide a means of linking bids.

1.2 The Results

The results can be summarized as in Figure 1. In the figure, R and Π represent seller revenue and bidder payoff respectively. Arrows point in directions of increase. Hence, for example, $R_C \geq R_D$. The figure implies the following:

(a) The case of the non-strategic bidder (Case A) provides the highest revenue of all cases, and the lowest payoff. That is, the use of any of: anonymity (Case C), anonymity and bid secrecy (Case D), and rationality (Case B), improves the non-strategic bidder's payoff and reduces seller revenue.

(b) Price discrimination – when the bidder is rational – is a disadvantage to the seller and an advantage to the bidder, when compared to consecutive auctions. That is, while comparing the case of both parties (seller and bidders) using information across stages, to that of neither party doing so, the former provides advantage to the bidder and disadvantage to the seller.

(c) The provision of anonymity for the non-strategic bidder decreases revenue, and the further provision of bid secrecy further decreases it.

(d) The decrease in revenue is not necessarily accompanied by a corresponding increase in payoff. Both (i) anonymity and (ii) anonymity and bid secrecy increase payoff and decrease revenue when the bidder is non-strategic. However, while anonymity increases payoff and decreases revenue, the further addition of bid secrecy decreases revenue but does not necessarily further increase payoff. In this case, the revenue decrease is not caused only by an increase in bidder payoff, it can also be caused by an opportunity loss for both bidder and seller (fewer sales because of higher prices).

(e) Anonymity is not as much of a disadvantage to the seller as is bidder rationality. Similarly, anonymity does not provide as much of an advantage to the bidder as does rationality. This is because anonymity provides information on individual valuations, while rationality does not.

(f) Anonymity and bid secrecy provide no advantage to the bidder when $k = 1$. This is because the auction exposes the second-highest valuation through the sale price.

(g) Rationality and consecutive auctions (no price discrimination) both provide higher payoff and lower revenue than price discrimination with a non-strategic bidder even when $k = 1$.

(h) Privacy protection (Cases C and D) does not always provide an advantage to the bidder when compared to the situation where neither bidders nor seller use information across stages (Case Ω).

(i) Privacy protection benefits higher bidders more than it does lower bidders. (This is not illustrated in Figure 1).

2. RELATED WORK

There is considerable literature on the use of information to improve strategies in multi-stage games, in particular in repeated auctions. For example, the revelation of the winning bid provides valuable information when there are sequential, repeated auctions for similar items [7]. Rational bidders anticipate the availability of information and bid lower and have higher payoffs. [14] examines the second-chance offer when there are two bidders. In this game, the only equilibria are mixed equilibria and value revelation is a sub-optimal strategy. Experimental results showed that the two-stage game generated more revenue than a sequential auction, i.e. that real bidders did not behave rationally.

Differential pricing has been practiced in the past, notably by Amazon [3], who had to stop the practice because of the associated negative publicity. [3, 13] have argued that vendors would be motivated to reduce consumer privacy in order to improve the accuracy of price discrimination. There is a group of interesting papers on what direction markets are expected to take with respect to the value of personal information, and the ease with which it is obtainable [13, 15, 3]. In particular, [3] describes how price discrimination is a suboptimal strategy for the seller.

Cryptographic auction schemes tend to provide exceptionally strong bid secrecy [5, 12], bidder anonymity [6], and correctness of the auction result [12, 5, 6, 4]. However, they are not yet widely deployed. For example, the FCC has implemented a new automated auction system [10] with neither encryption (except a Secure ID card for authentication purposes), nor anonymous protocols. Thus there is a need to examine the economic payoff of the use of cryptographic schemes in auctions, and to compare the results with those of the use of rational agents.

3. PRELIMINARIES

In this section we provide a short review of the open cry rising price auction, and add to the notation introduced in section 1. We end with an analysis of Case Ω, the sequence of $k+1$ auctions without price discrimination. As far as possible, we follow the notation of Krishna [11].

3.1 Notation

Stage I, when played by itself, is often referred to as the English Auction (EA). In an EA, a bidder with valuation x raises her bid b till she (a) wins the auction or (b) $b = x$. The highest bid of bidder i is denoted b_i, and her valuation x_i; the subscripts also denote order, so, for example, x_2 is the second-highest valuation:

$$b_i = \begin{cases} x_i & x_i < x_1 \\ x_2 + \delta & x_i = x_1 \end{cases} \tag{1}$$

where δ is the smallest increase allowed in the auction; we shall henceforth ignore δ.

The payoff to a bidder is the difference between x and the sale price if the bid is won, and zero if it is lost. If b denotes the highest bid of the bidder, the payoff Π is:

$$\Pi_{EA}(x) = \begin{cases} 0 & b \neq b_1, \quad i.e.\ x \neq x_1 \\ x_1 - b_1 = x_1 - x_2 & else \end{cases}$$

The sale price is the second highest valuation, i.e. $b_1 = x_2$ (see (1)). Because the probability distribution of the valuations is uniform, the expected value of the i^{th} highest valuation is [14]:

$$E[x_i] = \omega(1 - \frac{i}{N+1}) \tag{2}$$

where the expectation operator is denoted by $E[.]$. The expected revenue to the seller in Stage I is the expected value of the second-highest valuation, x_2:

$$E[R_{EA}] = E[b_1] = E[x_2] = \omega \times \frac{N-1}{N+1} \tag{3}$$

where R denotes revenue, and a subscript of I or II on Π or R denotes the value for Stage I or II respectively.

The expected value of the payoff to a bidder with valuation x is the expected value over (a) winning and losing the auction, and (b) all possible highest values of b_1 if the sale is won, and can be shown to be [11]:

$$E[\Pi_{EA}(x)] = \frac{x^N}{N} \tag{4}$$

3.2 Case Ω: Neither Seller nor Bidders Use Information

From the facts in the previous section, we may derive some simple results for the case of $k+1$ consecutive independent auctions without price discrimination.

Theorem 1. *In Case Ω the following are true:*

a. The price paid by bidder with valuation x is:

$$P_\Omega(x) = x_{i+1} \qquad x = x_i; i \leq k+1$$

b. The payoff is:

$$\Pi_{i,\Omega} = \begin{cases} x - x_{i+1} & x = x_i; i \leq k+1 \\ 0 & else \end{cases}$$

c. The seller's total revenue in Stages I and II is:

$$R_{I,II,\Omega} = \sum_{i=2}^{k+2} x_i$$

and its expected value is:

$$E[R_{I,II,\Omega}] = \omega \frac{k+1}{N+1}(N - \frac{k}{2} - 1)$$

Proof:

(a), (b) are straightforward and follow from (3). (c) follows from (2).

⊔

4. CASES A AND B: PRICE DISCRIMINATION AND NO PRIVACY PROTECTION

First we examine the case when the seller sees all the bids and the corresponding bidders, and can approach each individually, i.e. there is no privacy protection for the bidder. This information is not available to anyone else. As mentioned earlier, this case is very similar to eBay's current private auction. We examine the bidder payoffs and seller strategies for the non-strategic bidder, i.e. Case A. We then derive the strategy of the rational bidder, i.e. Case B, and then compare the payoffs and revenues of the two cases – both among themselves, and with Case Ω. Finally, we define the privacy cost of Case A over Case B.

4.1 The Non-strategic Bidder, Case A

In this section, we consider the non-strategic bidder of Case A who ignores the possibility of Stage II entirely. She bids as in an EA, and the seller determines the prices $P_1, ..P_k$ at which to offer the k additional objects to Bidders 2 through $k+1$.

Theorem 2. *In Case A, the following are true:*

a. The Stage II price offered to the i^{th} bidder is her highest failed bid, b_i.

b. The Stage II payoff and expected payoff to each bidder are zero.

c. The expected total payoff of Stage I and Stage II is that of the English Auction, i.e. that of Stage I:

$$E[\Pi_{I,II,A}(x)] = E[\Pi_{EA}(x)] = \frac{x^N}{N}$$

Table 1: Highest Bids for Auction of *Annexation Drone*

Bid Order	eBay Identity	Bid Value
b_1	erelfir	13.50
b_2	les-litwin	13.00
b_3	daredevilgo	5.55
b_4	jan1923	5.09

d. The seller's revenue in Stage II is $\sum_2^{k+1} b_i = \sum_2^{k+1} x_i$*, and its expected value is:*

$$E[R_{II,A}] = \frac{\omega k}{N+1}(N - \frac{(k+1)}{2})$$

e. The seller's total revenue in Stages I and II is:

$$R_{I,II,A} = R_{EA} + R_{II,A} = x_2 + \sum_2^{k+1} x_i$$

and its expected value is:

$$E[R_{I,II,A}] = E[R_{EA}]+E[R_{II,A}] = \frac{\omega}{N+1}((N-\frac{k}{2})(k+1)-1)$$

Proof: The seller is a monopoly and hence charges as much as he can. He knows the bidder is non-strategic, and hence that her highest failed bid is her valuation (1). Hence he charges her this value. The expression for the payoff (c) comes from (4), and that for the expected revenues, (d,e) is obtained by using (2). ⊔

Consider an example, the eBay auction for the trading card *Annexation Drone* [1]. The highest bids of individual bidders are listed in Table 1. Suppose the bidders are non-strategic (note that this is simply an illustrative example, we do not have the data to determine whether the bidders are non-strategic or rational), privacy protection as in Case A, i.e. the seller and only the seller has all the information of Table 1, and the seller has a total of four objects. Then he will offer the card for \$13 to les-litwin, \$5.55 to daredevilgo, and \$5.09 to jan1923. If all of them accept the offer, the seller's revenue is \$23.64. Notice that $k > \frac{N}{2}$, i.e. this example does not satisfy our requirement of $k < \frac{N}{2}$. This is easily rectified by assuming the above are the four highest bids in the auction, and that four other bids are not listed. The properties illustrated using this example, all through the paper, are not affected by this assumption.

Corollary 1. *The seller's total revenue in Case A is greater than that in Case* Ω*, and the bidder's payoff smaller, i.e.*

$$R_A \geq R_\Omega$$

$$\Pi_\Omega \geq \Pi_A$$

Proof: Straightforward from Theorems 1 and 2.
⊔

This means that, if only the seller uses information across stages, the bidder's payoff will reduce, and the seller's increase, when compared to the case where neither uses information across stages.

4.2 The Rational Bidder, Case B

In this section we consider Case B, of the rational bidder who takes into consideration the possibility of Stage II. This section examines her strategy, describes how the P_i are determined, provides expressions for the corresponding bidder payoffs and seller revenues, and compares these.

Theorem 3. *In Case B,*

a. The highest bid of a bidder with valuation x in Stage I is:

$$b_B(x) = \begin{cases} x_{k+1} & x \geq x_{k+1} \\ x & else \end{cases}$$

b. The price charged by the seller in Stage II is the same to all bidders who bid at least x_{k+1}*, as they are indistinguishable:*

$$P_{i,B} = b_1 \qquad i \leq k+1$$

c. The payoff is:

$$\Pi_B(x) = \begin{cases} x - x_{k+2} & x \geq x_{k+1} \\ 0 & else \end{cases}$$

d. The seller's total revenue in Stages I and II is:

$$R_{I,II,B} = (k+1)x_{k+2}$$

and its expected value is:

$$E[R_{I,II,B}] = (k+1)E[x_{k+2}] = (k+1)\omega\frac{N-k-1}{N+1}$$

e. The difference between the revenue in Case Ω *(consecutive auctions) and Case B is:*

$$R_{I,II,\Omega} - R_{I,II,B} = \sum_{i=2}^{k+1}(x_i - x_{k+2}) \geq 0$$

f. The expected difference between the revenues in Case Ω *and Case B is:*

$$E[R_{I,II,\Omega}] - E[R_{I,II,B}] = \omega\frac{k+1}{N+1} \times \frac{k}{2}$$

Proof Sketch:

(a). A bidder with valuation x will keep bidding till she is sure she has the item, but no longer. Hence she will stop increasing her bid once she is among the top $k+1$ bidders.

(b). The seller sees $k+1$ bidders with similar bids, all just greater than x_{k+2}. Let this highest bid value be b_1. If he cites a price P greater than b_1, he will lose some bidders whose valuations fall between b_1 and P. The expected value of the loss will be strictly greater than the gain obtained from increasing the price, as long as k is small enough ($k < \frac{N}{2}$). If he cites a price P smaller than this bid value, he will make a smaller revenue from k sales than he will from price b_1.

(c) Straightforward.

(d,f) Follow from (2).

(e) Follows from Theorem 1.

⊔

4.3 Summary: Rational and Non-strategic Bidders compared to Consecutive Auctions

Table 2 summarizes the prices, revenues and payoffs for Cases A and B.

Corollary 2. *Price discrimination does not benefit the seller when the bidder is rational:*

$$R_\Omega \geq R_B$$

$$\Pi_B \geq \Pi_\Omega$$

Corollary 3. *Rationality decreases revenue and increases payoff:*

$$R_A \geq R_B$$

$$\Pi_B \geq \Pi_A$$

This means that when both seller and bidders use information across stages, the bidders benefit while the seller is at a disadvantage, when compared to the situation of both parties not using any information across stages.

4.4 Privacy Cost

In this section we define the (comparative) privacy cost of one strategy over another and examine the privacy cost of not behaving rationally in Case A.

Definition *The privacy cost of Case* Δ *over Case* Λ*, to a bidder with valuation* x*, is the difference between the payoffs of* Λ *and* Δ*:*

$$\Phi(\Delta, \Lambda) = \Pi_\Lambda(x) - \Pi_\Delta(x)$$

A positive value of $\Phi(\Delta, \Lambda)(x)$ implies that Δ provides less privacy than does Λ to the bidder with valuation x.

An optimal strategy obviously provides the highest expected payoff and hence the lowest expected privacy cost when compared to any other strategy. Hence the privacy cost of Case A over Case B is obviously non-negative. The interesting question is, by how much?

The privacy cost of Case A over Case B is

$$\Phi(A,B)(x) = \begin{cases} x_2 - x_{k+2} & x = x_1 \\ x - x_{k+2} & x_1 > x > x_{k+1} \\ 0 & else \end{cases}$$

This means that, as expected, when the seller price discriminates, the payoff is smaller when the bidders behave non-strategically than when they behave rationally.

The privacy cost of Case A over Case Ω is

$$\Phi(A,\Omega)(x) = \begin{cases} x_i - x_{i+1} & x_1 > x = x_i > x_{k+1} \\ 0 & else \end{cases}$$

This means that the payoff when the seller price discriminates, and the bidders behave non-strategically, is smaller than when neither party uses information across stages.

Finally, the privacy cost of Case Ω over Case B is:

$$\Phi(\Omega,B)(x) = \begin{cases} x_{i+1} - x_{k+2} & x > x_{k+2} \\ 0 & else \end{cases}$$

This means that the payoff is larger when the seller price discriminates and the bidder behaves rationally than when neither uses information across stages. Hence, when the seller uses information across stages, if the bidder does too, the bidder is at an advantage over neither using information.

5. CASES C AND D: PRIVACY PROTECTION FOR THE NON-STRATEGIC BIDDER

In this section we examine the economic advantage, to the non-strategic bidder, of two specific types of privacy protection: anonymity and the combination of anonymity and bid secrecy. As in the previous section, quantitative expressions for bidder payoff, seller revenue and privacy cost are derived.

5.1 Case C: Anonymity

In this case, the seller knows the bids but not the corresponding bidder identities, cannot contact bidders individually, and may contact them as a group exactly once after the auction. He hence uses the known bids to estimate a best uniform price to offer to all bidders.

Theorem 4. *In Case C, the price offered, bidder payoffs and seller revenue are as follows:*

$$P_{i,C} = b_{i_0} \quad for\ some\ i_0 \in \{2, 3, ...k+1\}$$

$$\Pi_{II,C}(x) = \begin{cases} x - x_{i_0} & x \geq x_{i_0} \\ 0 & else \end{cases}$$

$$R_{II,C} = (i_0 - 1)x_{i_0}$$

$i_0 - 1 \leq k$ items are sold in Stage II. When $k = 1$, $i_0 = 2$.

Proof: $k = 1$. Obvious, the price offered is $b_2 = x_2$.

General k. The seller's revenue when the item is offered at a single price b_i is $(i-1)b_i$, as there are $i-1$ remaining bidders who would be willing to pay b_i for the item. (If the price is between bid values, the bidder loses no potential buyers – and increases his revenue – by taking it to the next highest bid. Hence we assume that the offered price is indeed the value of a bid). The seller runs through all possible offer prices $P = b_{i_0}$, and chooses the one that maximizes $(i-1)b_i$ conditional to $i - 1 \leq k$.

The payoff to bidder i, $i \geq i_0$ is zero (the bidders for $i > i_0$ cannot afford the item, and bidder i_0 pays her valuation) and other bidders have a non-zero payoff of value $x_i - x_{i_0}$. The seller's total revenue and the number of items sold follows.

Corollary 4.

a. The revenue in Case A is never smaller than that of Case C: $R_A \geq R_C$.

b. It is strictly larger, $R_A > R_C$ when $k \neq 1$, $\exists\ i\ s.\ t.\ b_i > b_{i_0}$, or $\exists\ i > i_0\ s.\ t.\ x_i > 0$.

c. $Pr[R_A > R_C] = 1$ when $k \neq 1$.

Proof: As the revenues are clearly equal when $k = 1$, we consider other values of k. Subtracting the revenue of Case C from that of Case A gives:

$$R_{II,A} - R_{II,C} = \sum_{i=2}^{k+1} x_i - (i_0 - 1)x_{i_0} = \sum_{i=2}^{i_0-1}(x_i - x_{i_0}) + \sum_{i=i_0+1}^{k+1} x_i \quad (5)$$

As $x_i \geq x_{i_0}$ when $i < i_0$, and $x_i \geq 0\ \forall i$, the above expression is non-negative. Further, if $\exists\ i\ s.\ t.\ b_i > b_{i_0}$ (i.e. $x_i > x_{i_0}$), or $\exists\ i > i_0\ s.\ t.\ x_i > 0$, at least one of the terms in the expression is non-zero and the entire expression is positive. Further, the probability of this not being true is zero. $\sqcup$

The first term in (5) represents the loss of revenue that contributes to the non-zero payoff of the bidders whose valuations are larger than x_{i_0}. The second term corresponds to the revenue lost because of bidders who could not afford the item at x_{i_0}, i.e. it represents the opportunity loss of the seller and the bidders with valuations smaller than x_{i_0}. Thus privacy protection provides benefit to the higher bidders, and could even be disadvantageous to lower bidders.

Consider the example of Table 1. Table 2 shows the data available to the seller in Case C. The seller's maximum revenue is: $Max(1 * 13, 2 * 5.55, 3 * 5.09) = 3 * 5.09 = 15.27$, and all three remaining bidders get the card at a price of \$5.09. Figure 2 shows the example set of bids. The rectangles under the bids represent the revenue in Case A. The revenue in Case C is the solid-colored rectangle. The loss in revenue in this case consists only of bidder payoff, as there is no opportunity loss.

Table 3: *Drone* Bids as Seen by Seller, Case C

Bid Order	Bid Value
b_1	13.50
b_2	13.00
b_3	5.55
b_4	5.09

Figure 3 illustrates with an example where revenue loss consists of both bidder payoff and opportunity loss, because the asking price for Stage II is strictly greater than the lowest bid.

Corollary 5.

a. The revenue in Case C is never smaller than that of Case B: $R_C \geq R_B$.

b. It is strictly greater, $R_C > R_B$, *when* $k \neq 1$ *and* $b_{i_0} \neq b_{k+1}$.

Proof: In Case C, the seller has access to more information about individual bids than in Case B, where he does not have access to the valuations of the higher bidders. Hence, his estimate of a price in Case C, b_{i_0}, is more accurate and hence provides at least as much revenue as the use of b_{k+1} as a price in Case B. Because the seller can use b_{k+1} as a price in Case C, the revenue of Case C is strictly greater exactly when he does not, i.e. when $b_{i_0} \neq b_{k+1}$.
$\sqcup$

Corollary 6.

a. The payoff in Case C is never larger than that of Case B: $\Pi_C \leq \Pi_B$.

b. It is strictly smaller, $\Pi_C < \Pi_B$, *when* $k \neq 1$ *and* $b_{i_0} \neq b_{k+1}$.

Proof: As in Corollary 5.

5.2 Case D: Anonymity and Bid Secrecy

In this case, the seller can contact all the bidders as a group with a single offer, but has no information on the bids except on the winning one. Suppose that, at price p, $f(p)$ buyers are willing to buy the item. Then

$$f(p) = i - 1, \quad b_{i+1} < p \leq b_i$$

In Case C, the bidder possesses the exact values of $f(p)$ because he knows b_i, and he can hence maximize the exact value of his revenue $(pf(p))$. In Case D, however, he does not know b_i and would choose the value of p that maximizes his expected revenue, $pE[f(p)]$.

Theorem 5. *In Case D, the offer price, expected number of items sold, bidder payoffs and seller revenue are as follows:*

$$P_{i,D} = \frac{b_1}{N-2} \times (N - k - 1)$$

where k *is the expected number of items sold.*

$$\Pi_{II,D}(x) = \begin{cases} x - P_{i,D} & x \geq P_{i,D} \\ 0 & else \end{cases}$$

$$E[R_{II,D}] = \omega \frac{(N-1)(N-k-1)}{(N+1)(N-2)}$$

Proof Sketch: Given a price p, the expected number of buyers is: $E[f(p)] = (1 - \frac{p}{b_1})(N - 2) + 1$. The expected revenue at that price is hence $p[(1 - \frac{p}{b_1})(N - 2) + 1]$, the area of a rectangle under the line. The maximum expected revenue is the maximum area of a rectangle under the line, provided the value of $E[f(p)]$ is not larger than k. The expected revenue increases with a decrease in p till $E[f(p)] = \frac{N}{2}$, hence the price should be the one where $E[f(p)] = k$.
$\sqcup$

Corollary 7. *The revenue in Case C is never smaller than that of Case D.*

$$R_C \geq R_D$$

Proof: The price in Case C is also a single price offer to the group of bidders as a whole. However, the single price offer in Case C depends on the actual bids, and maximizes the revenue for those bids. Hence the revenue obtained in Case D for the same bids cannot be greater than that in Case C. $\sqcup$

Consider the example of Table 1. In Case D, the seller knows only the winning bid, \$13.50, and the number of remaining bidders, 3. The Stage II offer is \$10.13. At this price only one bidder can buy the item, and the seller's revenue is \$10.13. Figure 4 illustrates the differences between the revenues in Cases A and D. Also shown is the estimated straight line representing the price at which i buyers would buy an item (the line $f(p)$ described above is the inverse of this line).

Notice that the optimal price for Case D (\$ 10.13) is higher than the price dictated by knowledge of the bids (i.e. the price of Case C, \$5.09), hence there is lost revenue in the form of an opportunity loss for bidders who are not able to afford the item. The larger price does not make up for this opportunity loss, and the revenue for the seller in Case D is not greater than that in Case C. It would also be possible to obtain a price in Case D that is lower than dictated by knowledge of the bids. In such a situation, the revenue loss goes at least partly towards increasing bidder payoffs, and might also result in more sales. This situation is not illustrated.

While it is clear that the revenue in Case D is not larger than that in Case C, the payoffs in Case D may be smaller than those in Case C. As demonstrated in Figure 4, the revenue loss does not always go towards bidder payoff (the payoffs of bidders in the *Drone* example are smaller in Case D than in Case C). An examination of the difference in expected payoffs is beyond the scope of this paper.

5.3 Summary: Cases C and D

Table 4 summarizes the prices, revenues and payoffs for Cases A, C and D.

The privacy cost of Case A over Case C is zero for $k = 1$, and, for other values of k, it is non-negative:

$$\Phi(A,C)(x) = \Pi_C(x) - \Pi_A(x) = \begin{cases} x - x_{i_0} & x \geq x_{i_0} \\ 0 & else \end{cases} \quad (6)$$

i.e. $N - i_0 + 1$ of the bidders see no difference in the two cases. In the *Drone* example, $\Phi(A,C)$ is \$7.91, \$0.46, 0, for the three losing bidders respectively, and the lowest bidder sees no effect of the privacy provided.

Similarly, the privacy cost to the bidder of Case A over Case D is zero for $k = 1$, and, for other values of k, it is non-negative:

$$\Phi(A,D)(x) = \Pi_D(x) - \Pi_A(x) = \begin{cases} x - P_{i,D} & x \geq P_{i,D} \\ \\ 0 & else \end{cases} \quad (7)$$

In the *Drone* example, $\Phi(A,D)$ is \$2.87, 0 and 0, for the three losing bidders respectively, and the lowest two bidders do not see the impact of the privacy provided.

It is not possible to say if $\Phi(C,D)$ and $\Phi(\Omega,D)$ are positive or negative, i.e., in general, they may be either. An examination of the expected privacy costs, $E[\Phi(C,D)]$ and $E[\Phi(\Omega,D)]$, is beyond the scope of this paper.

6. CONCLUSIONS

This paper models the second chance offer as a two-stage game consisting of an English Auction followed by a price discrimination stage. It examines the effect of privacy protection techniques and the use of rational agents on strategies, payoffs, and revenue. It shows that certain price discrimination is a suboptimal strategy for the seller when the bidder is rational. Further, it shows that privacy protection is of no use to the non-strategic bidder when there is only one extra item to sell in the price discrimination stage. However, when there is only one additional item to sell in the second stage, rationality does provide an advantage to the bidder.

Both privacy protection and the use of rational agents result in an increase in bidder payoff. Increasing the extent of privacy protection – from only anonymity to anonymity and bid secrecy – decreases revenue. However, the lost revenue may correspond to lost opportunity and does not always increase bidder payoff when the bidder is non-strategic. The increase in payoff due to privacy protection is generally experienced by higher bidders. Lower bidders tend to experience opportunity loss.

The paper defines the privacy cost of one case over another as the difference in payoffs between the two cases. It opens up a number of possibilities for future research – what is the privacy cost when price discrimination is performed for similar items and not for the same one? What are optimal strategies for rational agents who are aware of the possibility of such price discrimination? How does seller strategy – in the form of price discrimination at random, with probability $\alpha \in (0,1)$ – affect payoffs and bidder strategies? In what way does the existence of privacy cost affect some of the fundamental results in auction theory, such as truth revelation in second-price sealed-bid auctions, and revenue equivalence? In what way does the existence of privacy cost affect the general results in game theory? The authors are examining some of these questions in manuscripts in preparation [8, 9].

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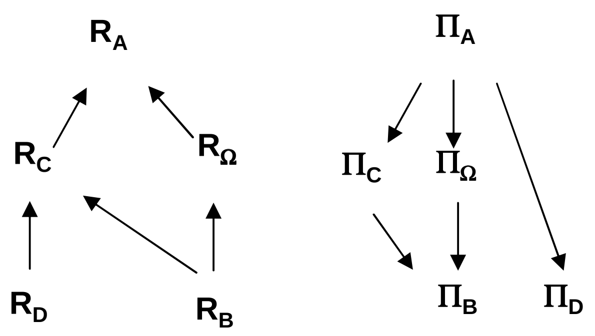

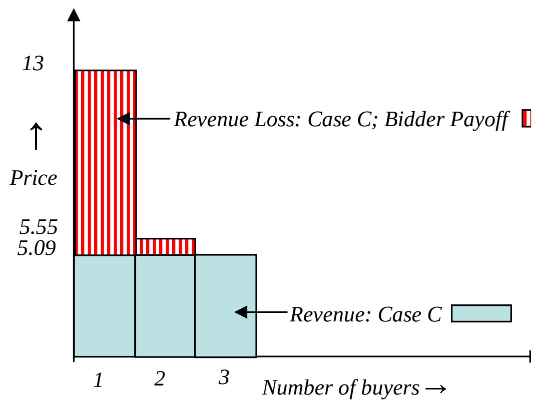

Figure 2: The Stage II Revenue for Case C, t
card *Annexation Drone* – revenue loss due to
payoff

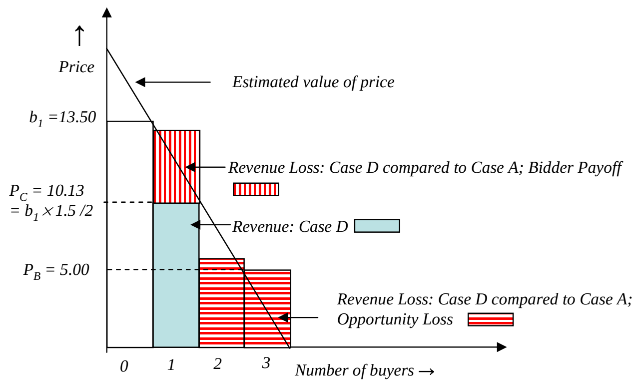

Figure 4: The Revenues for Cases A and D for *Annexation Drone*

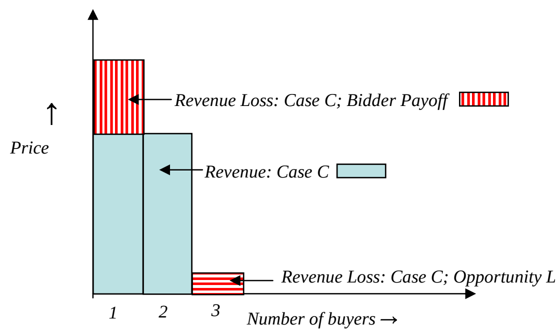

Figure 3: Example of Stage II Revenue wit
opportunity loss and revenue loss due to bidd
off

Table 2: Rational and Non-strategic Bidders, Cases A and B: Prices, Payoffs and Revenues for Stages I and II.

	Consecutive Auctions	Non-strategic	Rational $\alpha = 1$
Price P_i	$\begin{cases} x_{i+1} & i \le k+1 \\ 0 & else \end{cases}$	x_i	$\begin{cases} x_{k+2} & i \le k+1 \\ 0 & else \end{cases}$
$\Pi(x)$	$\begin{cases} x - x_{i+1} & x = x_i \ge x_{k+1} \\ 0 & else \end{cases}$	$\begin{cases} x_1 - x_2 & x = x_1 \\ 0 & else \end{cases}$	$\begin{cases} x - x_{k+2} & x \ge x_{k+1} \\ 0 & else \end{cases}$
R	$\sum_{i=2}^{k+2} x_i$	$x_2 + \sum_{i=2}^{k+1} x_i$	$(k+1)x_{k+2}$
$E[R]$	$\omega \frac{k+1}{N+1}(N - \frac{k}{2} - 1)$	$\omega \frac{k+1}{N+1}(N - \frac{k}{2} - \frac{1}{k+1})$	$\omega \frac{k+1}{N+1}(N - k - 1)$

Table 4: Non-strategic Bidder: Prices, Payoffs and Revenues for Stage II

	k=1 A, C, D	k Case A	k Case C	k Case D $P = \frac{b_1}{N-2} \times N - k - 1$
Price P_i	b_2	b_i	b_{i_0}	P
Payoff $x - P_i$	0	0	$x - x_{i_0} \quad x \ge x_{i_0}$ $0 \quad else$	$x - P \quad x \ge P$ $0 \quad else$
Payoff in *Drone* example	0	0	\$7.91, \$0.46, 0	\$2.87, 0, 0
Revenue	x_2	$\sum_{i=2}^{k+1} x_i$ Expected Revenue = $\frac{\omega k}{N+1}(N - \frac{(k+1)}{2})$	$(i_0 - 1)x_{i_0} = \sum_{i=2}^{i_0} x_{i_0}$	$P \times k$ Expected Revenue = $\frac{\omega k}{N+1} \times \frac{N-1}{N-2} \times (N - k - 1)$
Revenue in *Drone* example	\$13.00	\$23.64	\$15.27	\$10.13