ON THE ACCURACY OF SCANNING COLOUR IMAGES

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Abstract

In order to reproduce perceived colour accurately the three bands of a colour image must be obtained using sensors whose relative spectral sensitivity satisfies certain constraints. This paper reviews these constraints, discusses the design of filters which satisfy these constraints and examines the effects of inaccuracies in the sensors responses. The mathematical techniques used for the error analysis have not been used previously in colour scanning filter design. The results of this study are applicable to most colour systems including television, lithography, copiers and printers.

INTRODUCTION

Most current research in colour systems assumes that the visual frequency spectrum can be adequately represented by samples taken about 10 nm apart over the range 400-700 nm. Integrals are approximated by summations, and a continuous function of wavelength is represented by an N-vector of its sampled values.

It had long been suspected that all visual colour stimuli might be recreated by linear combinations of three primary signals with particular properties. This was later confirmed when it was established that the human eye contains three types of colour sensors, known as cones. The spectral sensitivity of each of the cones determines human sensitivity to colour. The three types of cone responses correspond roughly to red (α), green(γ), and blue(β). If **S** is the Nx3 matrix whose columns represent the sensitivity functions of the three kinds of cones, and **f** is a radiant signal, the output of the cones can be modelled as :

$$\mathbf{c} = [c_1, c_2, c_3]^T = \mathbf{S}^T\mathbf{f} \quad (1)$$

Though the eye itself is not a linear system, the nonlinearity in the sensing process is always assumed monotonic in the intensity range of interest. In fact, the sensor response is usually modelled as a logarithmic function. Let $V(.)$ represent the nonlinearity, then the stimulus provided by a radiance signal **f** can be modelled as [1]:

$$\mathbf{v} = [V(c_1), V(c_2), V(c_3)]^T \quad (2)$$

Because $V(.)$ is monotonic and hence one-to-one, two radiance signals, represented by the N-vectors **f** and **g**, provide identical colour stimuli iff

$$\mathbf{S}^T\mathbf{f} = \mathbf{S}^T\mathbf{g} \quad (3)$$

Two such signals **f** and **g** are known as metamers. Since the aim of an accurate colour reproduction system is to reproduce a perceived colour, it is not necessary to obtain the entire spectrum of the signal to be reproduced. It is sufficient to obtain measurements that are in some sense equivalent to the three measurements made by the cones in the human eye, so that a metamer may be obtained.

2. CONSTRAINTS ON SCANNING FILTERS

The Commission Internationale de l'Eclairage (CIE) has established a set of functions equivalent to

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the sensitivity functions of the cones, known as the CIE colour matching functions. These functions have been determined experimentally. Let

$$\mathbf{P} = [\mathbf{p}_1 \; \mathbf{p}_2 \; \mathbf{p}_3] \qquad (4)$$

where $\mathbf{p}_i, i = 1, 2, 3$, are the three primaries used in the experiment. Let $\mathbf{e}_i$ denote the N-vector whose i^{th} component is unity, and all other components are zero. Let $\mathbf{b}_{ij}$ denote the amount of the j^{th} primary required to provide an exact colour match with $\mathbf{e}_i$. Then,

$$\mathbf{S}^T\mathbf{e}_i = \mathbf{S}^T\mathbf{P}[\mathbf{b}_{i1} \; \mathbf{b}_{i2} \; \mathbf{b}_{i3}]^T \qquad (5)$$

Hence,

$$\mathbf{S}^T = (\mathbf{S}^T\mathbf{P})\mathbf{B}^T \qquad (6)$$

The three columns of **B** are the CIE matching functions, shown in Fig. 1. A linear transformation of these functions yields three all-positive matching functions, $\mathbf{a}_i, i = 1, 2, 3$, shown in Fig. 2. Though these all-positive matching functions do not correspond to realizable primaries, they possess the advantage of being physically realizable as they possess no negative components. If **A** is the matrix whose columns are these functions, then [1]

$$\mathbf{S}^T\mathbf{f} = \mathbf{S}^T\mathbf{g} <=> \mathbf{A}^T\mathbf{f} = \mathbf{A}^T\mathbf{g} \qquad (7)$$

The perceived colour is defined by its tristimulus values with respect to the standard Nx3 filter set **A**,

$$\mathbf{t} = \mathbf{A}^T\mathbf{f} \qquad (8)$$

If the Nx3 matrix **P** represents the spectral radiance distribution of the set of three primaries corresponding to the set of filters **A**, then **Pt** is a metamer of **f**. Thus, the colour stimulus of **f** may be reproduced exactly if **t**, the vector of tristimulus values, is known and the primaries are realisable. The columns of **A** span the three dimensional space spanned by the sensitivity functions of the human eye. This space is known as the Human Visual Subspace (HVSS). In general, given a set of primaries, colour-matching functions may be defined as in (5). Colour matching functions that are based on other primaries will also span the HVSS, and hence will be equivalent to the cone sensitivities. Two signals provide identical colour stimuli iff their projections onto the HVSS are identical; thus, the projection onto the HVSS,

$$P_v(\mathbf{f}) = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{f} \qquad (9)$$

uniquely defines the perceived colour of **f**. This projection of **f** is known as the fundamental of **f**. Hence the constraint on any accurate colour sensing device is that the sensitivities of the sensors span the HVSS [2], so that the fundamental may be determined.

Colour reproduction begins with correctly determining the projection of a given spectrum onto the HVSS. The CIE matching functions are not the only basis that may be used for the HVSS. If the primaries corresponding to the set of scanning filters do not need to be realisable, the scanning filters can be any linearly independent set of three vectors from the HVSS. In fact, a set **M** may be used for the purpose of colour scanning iff

$$\mathbf{M} = \mathbf{A}\mathbf{X} \qquad (10)$$

where **X** is an invertible 3x3 matrix.

3. ILLUMINATION AND COLOUR SCANNING

The quality of colour reproduction is highly dependent on the recording and display illuminants. For example, the quality of a colour slide will be different from that of a colour photograph (if for no other reason) because the display illuminants are different. If **D** is a diagonal matrix whose diagonal elements are the spectral values of the recording illuminant and **r** is the reflectance of an object, the recorded tristimulus values are

$$\mathbf{t} = \mathbf{A}^T(\mathbf{D}\mathbf{r}) = (\mathbf{A}_D)^T\mathbf{r} \qquad (11)$$

where

$$\mathbf{A}_D = \mathbf{D}\mathbf{A} \qquad (12)$$

Thus, the information obtained under the recording illuminant is not the projection of **r** in the space spanned by the columns of **A**, but in the space spanned by the columns of $\mathbf{A}_D$. This space will be referred to as the HVSS defined by the illuminant **D**. Similarly, the projection that is needed to accurately reproduce the colour of the object with reflectance **r** under the recording illuminant is the projection of **r** in the space defined as above by the recording illuminant. It is, in general, impossible to exactly reproduce the colour of $\mathbf{r}$ under a different illuminant with only the knowledge of its projection in the space defined by the recording illuminant. This is the basic problem in colour correction.

A colour stimulus can be accurately reproduced even if the recording and display illuminants are different if the filters are appropriately designed. Let $\mathbf{D}_1$ and $\mathbf{D}_2$ be diagonal matrices representing the recording and display illuminants respectively. The designed filters, $\{\mathbf{m}_i\}_{i=1}^3$ must be such that the space spanned by $\{\mathbf{D}_1\mathbf{m}_i\}_{i=1}^3$ is the same as the space spanned by $\{\mathbf{D}_2\mathbf{a}_i\}_{i=1}^3$. The information obtained by scanning is the projection of the signal onto the Human Visual Subspace defined by the illuminant $\mathbf{D}_2$.

Now suppose a reflectance spectrum $\mathbf{g}$ is required such that $\mathbf{g}$ is a metamer of $\mathbf{f}$ under more than one display illuminant. For example, a colour reproduction is sought that matches $\mathbf{f}$ in daylight as well as under fluorescent lighting. In such a situation, in general, six parameters are required to characterise the signal $\mathbf{f}$ so that accurate colour reproduction is possible. These six parameters together represent the projection of the signal $\mathbf{f}$ onto the six-dimensional space defined by the two display illuminants. Hence, it may sometimes be essential to use more than three filters to ensure accuracy in colour reproduction.

4. CONSTRUCTION OF SCANNING FILTERS

An important criterion in the design of scanning filters is their realisability as optical filters. For this reason, all scanning filters $\mathbf{m}$ must belong to the set:

$$C_n = \{\mathbf{f}|\mathbf{f}(i) \geq 0\} \qquad (13)$$

A popular way of realising a designed scanning filter is to construct it from a set of existing optical filters. Such sets are commercially available. Suppose a scanning filter, $\mathbf{m}$, is to be obtained from a set of n existing filters. Let $\{\mathbf{r}_i\}_{i=1}^n$ denote the set of the n given filters where $\mathbf{r}_i$ (i=1, 2, ...n) is a vector in N-dimensional transmission space. If it is possible to express $\mathbf{m}$ as a product of these filters, $\mathbf{m}$ must belong to the set C_T in transmission space [1]:

$$C_T = \{\mathbf{f}|\mathbf{f} = \alpha \prod_{i=1}^{n} \mathbf{r}_i^{u_i},\ \alpha,\ u_i \geq 0\} \qquad (14)$$

where u_i is a scalar representing the density of the optical filter $\mathbf{r}_i$ in the construction of $\mathbf{m}$, and α is a scalar multiplying factor. The HVSS may be represented by the set

$$C_v = \{\mathbf{f}|\mathbf{f} = \mathbf{A}\mathbf{x},\ for\ \mathbf{x}\ any\ 3-vector\} \qquad (15)$$

All scanning filters designed and reported in the literature to date have been designed to belong to C_v and C_n. Every constructed approximation, in turn, belonged to C_T, which itself is a subset of C_n. These approximations were all, in some sense, 'close' to the designed filter $\mathbf{m}$, but none were accurately constructed by the designers, hence it is unlikely that any were actually in C_v. It is clear that a set of scanning filters constructible in product form must lie in the intersection of C_T, C_n, and C_v. Filters may also be designed as sums, sums-of-products and products-of-sums of $\{\mathbf{r}_i\}_{i=1}^n$. Whether it is possible to construct a set of suitable scanning filters depends on the set, $\{\mathbf{r}_i\}_{i=1}^n$, and it is possible that filters that lie in C_v cannot be constructed from the given set. In such a situation, it is not possible to construct three filters that span the HVSS, though it might be possible to construct four filters such that the span of the four filters includes the HVSS.

It is clear that it is often impossible to construct a filter set that spans the HVSS. An error analysis is essential in such a situation and provides insight into the effect of filter errors on colour reproduction errors.

5. EFFECT OF ERRORS ON COLOUR REPRODUCTION

An error in the construction of a set of scanning filters that are designed to span the HVSS could change the space spanned by the filters. From Sections 3 and 4 it is clear that such errors do occur in practical situations, and that such errors will complicate further the already difficult task of colour correction. Let $\mathbf{M}$ denote the set of scanning filters. The information obtained from this set is

$$P_M(\mathbf{f}) = \mathbf{M}(\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\mathbf{f} \qquad (16)$$

where $P_M(.)$ denotes the projection operator onto the space spanned by the scanning filters. The fundamental of $P_M(\mathbf{f})$ is:

$$P_v(P_M(\mathbf{f})) = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{M}(\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\mathbf{f} \qquad (17)$$

The difference between this and the fundamental of $\mathbf{f}$ may be defined as the error in the reproduction. This error is then:

$$\mathbf{e} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T(\mathbf{I} - \mathbf{M}(\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T)\mathbf{f} \qquad (18)$$

which may be rewritten as:

$$\mathbf{e} = P_v(I - P_M)(\mathbf{f}) \qquad (19)$$

The mean-square error may be seen to be:

$$E[||\mathbf{e}||^2] = Trace((P_v)^T(I - P_M)\mathbf{R}(I - P_M)P_v \qquad (20)$$

where

$$\mathbf{R} = E[\mathbf{f}\mathbf{f}^T] \qquad (21)$$

For a given sample set this error is easily computed. This error measure may be used to define a measure of goodness [3] of the set of scanning filters. Minimizing the error corresponds to a familiar least-squares problem. Error minimization may be a possible way of obtaining an optimal constructible set of scanning filters, where optimal corresponds to 'most accurate' with respect to the error measure defined above.

6. CONCLUSIONS

In conclusion, the problem of designing optimal scanning filters can be posed as a signal processing problem, where standard signal processing techniques may be used to advantage. Optimization techniques may be used towards solving the problem if the problem is viewed in the discrete signal setting, as proposed. The effect of filter errors on the colour reproduction is a typical form of a signal processing problem.

References

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[3] VORA, P. L., and TRUSSELL, H. J., *On Measures of Goodness of a Set of Colour Scanning Filters*, Proceedings, SPIE/IS&T Symposium on Electronic Imaging: Science and Technology, Feb. 1992.

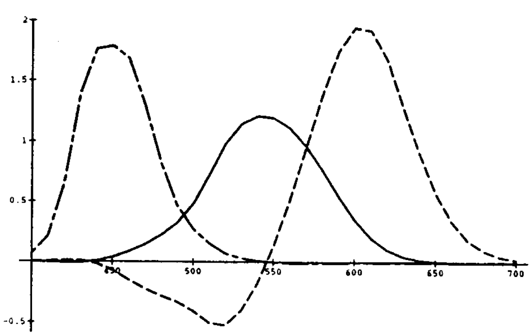

FIG. 1
(EXPERIMENTAL) CIE MATCHING FUNCTIONS

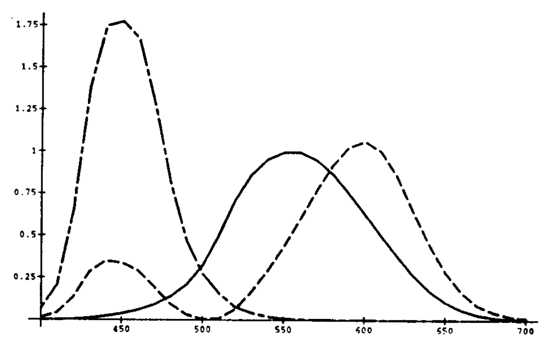

FIG. 2
ALL-POSITIVE CIE MATCHING FUNCTIONS