Mathematical Methods for the Design of Color Scanning Filters

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***Abstract*—The problem of the design of color scanning filters is addressed in this paper. The problem is posed within the framework of the vector space approach to color systems. The measure of the goodness of a set of color scanning filters presented in earlier work is used as an optimization criterion to design color scanning filters modeled in terms of known, smooth, nonnegative functions. The best filters are then trimmed using the gradient of the mean square ΔE_{ab} error to obtain filters with a lower value of perceptual error. The results obtained demonstrate the utility of the method.**

I. Introduction

FILTERS used for multiband image recording for the purpose of color reproduction are referred to as color scanning filters even though many modern imaging devices such as charge-coupled device (CCD) arrays do not "scan." These filters are a basic component of many color reproduction systems. The goal of the color scanning process is to obtain a linear transformation of the Commission Internationale de L'Eclairage (CIE) tristimulus values [1]. For color correction applications, CIE tristimulus values for more than one viewing illuminant may be needed. For satellite applications, the "spectral signatures" may be desired. In any multiband image-recording problem, physical filters need to be designed and manufactured. In many instances the filters can be chosen from a bank of existing filters. In other cases, the filters are to be custom manufactured. The combined effect of the optical path, the recording illuminant and the detector sensitivity, which must also be taken into account, often complicates the design procedure.

This paper formulates the design of a set of three or more color scanning filters as an optimization problem. The optimization criterion is the measure of goodness ν developed in [14]. This criterion is different from those used by other researchers [3], [4], [6], [23] in that it measures the joint performance of the set of filters as a whole and not the performance of individual filters. Most of the literature in the design of color scanning filters reports optimization routines that minimize a norm of the difference between each constructed filter and the combined effect of the corresponding CIE matching function and viewing illuminant [3], [6], [23] or maximize the q-factor (defined by Neugebauer [10]) of the individual scanning filters [4]. The measure defined in [5] is the average of the q-factors of the individual filters and, hence, a measure of individual filter performance. There has been no reported research on the use of a measure of the entire set of color scanning filters as an optimization criterion.

In the case when the filters are to be chosen from a discrete set of filters, the problem of finding a filter set with maximum value of the measure ν may be solved by an exhaustive search. When the filters are to be fabricated (as interference filters, for example), problems of physical realizability lead to a parametrized optimization problem which may be solved using existing optimization algorithms. Filters designed thus may be trimmed using a perceptual error measure like the mean square ΔE_{ab} error over a data set. The method described in this paper has proved useful for colorimetric applications, as demonstrated in Section IV. It can also be used for the design of filters for other multiband image recording problems, specifically for the design of filters with applications in satellite imagery.

The method proposed can be used for any imaging system for which the sensor characteristic (defined as the combined effect of the lamps, light path and sensor characteristic) is known. It is dependent on accurate sensor characterization, as are all scanning filter design methods. It has been observed that, even in the absence of precise scanner characterization, this method provides results far superior to other methods using the same scanner characterization. The method assumes, in particular, that the continuous waveforms representing the viewing and scanning illuminants, the radiant spectrum to be measured, etc., have been sampled at a sufficiently high rate in the wavelength domain. This common assumption forms the basis of the vector space approach to color.

The method presented here does not restrict the number of samples per spectrum. The experimental results presented here assume the commonly accepted 10 nm sampling rate for illustration purposes and for comparison with other methods [7] that use this rate. This rate may not be sufficient to characterize, for example, fluorescent illuminants. The effect of insufficient sampling on color system design has been discussed in [12] and [13]. The sensitivity of filter performance to error in scanner characteristic measurement will not be addressed here.

It is often not possible to fabricate the designed "optimal" filters exactly. The fabricated filters will not have the specified

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transmissivities, and this perturbation in filter transmissivities leads to a general degradation of filter performance demonstrated by larger perceptual errors and smaller values of the optimization criterion ν. The problem of sensitivity of the measure ν and the perceptual error to errors in filter fabrication is the subject of a companion paper [17].

The measure ν used in this paper is based on Euclidean distances in $\mathcal{R}^N$ and is not a perceptual error measure. A common perceptual error measure is ΔE_{Lab} which is the Euclidean distance in the CIE $L^*a^*b^*$ space. The transformation from the spectral space of $\mathcal{R}^N$ to the CIE XYZ space is linear; however, the transformation from XYZ to $L^*a^*b^*$ is nonlinear [24]. Although the measure is not directly related to the perceptual error, it has been shown to give a good indication of the average ΔE_{ab} error over standard data sets [14]. Because the ultimate performance of the filters depends on perceptual error, the values of the ΔE_{ab} measure are included in the data reported in this paper to demonstrate the power of the design method. For this reason, let us define the terms here.

The color for a radiant spectrum is determined by

$$\mathbf{t} = \mathbf{A}^T\mathbf{r} \tag{1}$$

where $\mathbf{r}$ is an N-vector representing the sampled spectrum of a radiant source, $\mathbf{A}$ is an $N\times 3$ matrix representing the CIE color matching functions and $\mathbf{t}$ is a 3-vector representing the CIE tristimulus values. A reflectance spectrum can be measured by producing a radiant spectrum

$$\mathbf{r} = \mathbf{Lf}$$

where $\mathbf{f}$ is a reflectance spectrum and $\mathbf{L}$ is an $N \times N$ diagonal matrix representing the spectrum under which the reflecting object is viewed. The color matching functions and the illuminant can be combined in a single matrix, $\mathbf{A}_L = \mathbf{LA}$, which defines the human visual subspace (HVSS) under the illuminant $\mathbf{L}$.

Let $\mathbf{t}(\mathbf{f}) = \mathbf{A}_L^T\mathbf{f} = [x_f, y_f, z_f]^T$ be the actual CIE tristimulus values for the reflectance signal $\mathbf{f}$ for the viewing illuminant, and $\mathcal{F}[\mathbf{t}(\mathbf{f})] = [L_f^*, a_f^*, b_f^*]^T$ be the transformed (actual) tristimulus vector in CIELAB space. Let $\hat{\mathbf{t}}(\mathbf{f}) = [x, y, z]^T$ be the estimated CIE tristimulus values for the reflectance signal $\mathbf{f}$ given a particular viewing illuminant. Let $\mathcal{F}[\hat{\mathbf{t}}(\mathbf{f})] = [L^*, a^*, b^*]^T$ be the transformed (estimated) tristimulus vector in CIELAB space [24]. Then $\Delta E_{ab}(\mathbf{f})$, the ΔE_{ab} error for reflectance spectrum $\mathbf{f}$, is given by

$$\Delta E_{ab}(\mathbf{f}) = \sqrt{[(L^* - L_f^*)^2 + (a^* - a_f^*)^2 + (b^* - b_f^*)^2]}.$$

The error vector in CIELAB space is not linearly related to the corresponding error vector in tristimulus space and the average ΔE_{ab} error over a data set cannot be characterized by average, or individual, tristimulus error. The characterization of average ΔE_{ab} error over a data set will involve the actual calculation of ΔE_{ab} error at each point in the data set and is thus, data dependent. This average error is given by

$$E_{ave} = \frac{\sum_{\mathbf{f}} \Delta E_{ab}(\mathbf{f})}{n} \tag{2}$$

where $\sum_{\mathbf{f}}$ represents the sum over the n spectra in the data set. While it is possible to use E_{ave} as an optimization function, it is most unwieldy.

In this paper, Section II introduces notation and presents the motivation behind posing the filter design problem as constrained optimization. Section III presents ways of incorporating constraints of physical realizability into the problem, design procedures for designing a set of filters which may be fabricated and a method of trimming the parametrized optimal filters to get filters with optimal performance with respect to the average square ΔE_{ab} error over a data set. Section IV implements the parametrizations and the trimming method of Section III for particular scanner characteristics. Conclusions are presented in Section V.

II. Constrained Optimization Problem

The goal of color measurement is to determine the $3K$ values $[\mathbf{A}_{L_1}, \mathbf{A}_{L_2}, \cdots \mathbf{A}_{L_K}]^T\mathbf{f}$ where K is the number of viewing illuminants. When $K > 1$, the problem has applications in color correction [20], [21]. The problem of determining the spectral signatures of portions of the earth's surface may be expressed as the problem of determining $\mathbf{S}^T\mathbf{f}$, where the columns of $\mathbf{S}$ represent the responses of sensors used for remote sensing [2], [16]. The problem of color scanning may, hence, be generalized to the problem of obtaining the set of s values $\mathbf{t} = \mathbf{V}^T\mathbf{f}$. Here, $\mathbf{f}$ is an N-vector and $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots \mathbf{v}_s]$ where $\mathbf{v}_i$ may represent a CIE matching function for a particular viewing illuminant as in color scanning, or the function characterizing a sensor response in the satellite imaging problem. The vector $\mathbf{t}$ may be referred to as the s-stimulus vector.

Suppose the diagonal matrix $\mathbf{H}$ representing the scanner characteristic is known. Let $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \cdots \mathbf{m}_r]$ represent a set of r scanning filters. The matrix $\mathbf{HM}$ represents the scanning system or the set of effective scanning filters and is denoted $\mathbf{M}_H$. The scanning measurements are modeled by the r-vector $\mathbf{M}_H^T\mathbf{f}$. The scanning system need not replicate the columns of $\mathbf{V}$ as it is sufficient to obtain measurements from which the values $\mathbf{V}^T\mathbf{f}$ may be determined through a linear transformation. Thus, the filter design problem is formulated as one of finding a set of vectors which span a desired subspace, defined as the range space of $\mathbf{V}$. As it may not always be possible to fabricate a perfect set of filters, it is necessary to have a means of evaluating an imperfect set of filters. A measure of a set of filters may be used as an optimization criterion for filter design.

A. The Data-Independent Measure ν

The measure, ν of a set of scanning filters [14] is used as an optimization criterion in this paper. This measure is based on minimizing the mean square error of spectra that are independent and identically distributed (i.i.d.) at each wavelength [14], i.e., a data set whose autocorrelation matrix is a scalar multiple of the identity. While real data sets are usually correlated, the measure ν is data independent. Another motivation for the measure is that it measures the "distance" between two subspaces of $\mathcal{R}^N$, the range space of $\mathbf{M_H}$, defined by the scanning filters, and the visual space, the range space of $\mathbf{V}$, defined by the visual system. Thus, we write the measure as a function of these spaces, $\nu(\mathbf{V}, \mathbf{M_H})$. An increase in the measure ν generally corresponds to a decrease in average ΔE_{ab} error over common data sets [14], though there are exceptions.

In order to represent the problem of scanning filter design as an optimization problem, the measure ν of the scanning system may be expressed explicitly in terms of $\mathbf{M}$. The expression for ν in [14] is

$$\nu(\mathbf{V}, \mathbf{HM}) = \frac{\sum_{i=1}^{\beta} \lambda_i^2(\mathbf{O}^T\mathbf{N})}{\alpha}$$

where $\mathbf{N}$ and $\mathbf{O}$ are matrices whose columns form an orthogonal basis for the range of $\mathbf{HM}$ and $\mathbf{V}$, respectively, λ_i^2 represents the ith singular value, and α and β are the dimensions of the range space of $\mathbf{V}$ and $\mathbf{MH}$, respectively. Substituting for the singular values in the above expression gives

$$\nu(\mathbf{V}, \mathbf{HM}) = \frac{\text{Trace}\, \mathbf{N}^T\mathbf{O}\mathbf{O}^T\mathbf{N}}{\alpha} = \frac{\text{Trace}\, \mathbf{N}\mathbf{N}^T\mathbf{O}\mathbf{O}^T}{\alpha}$$

as Trace $\mathbf{XY}$ = Trace $\mathbf{YX}$. $\mathbf{OO}^T$ represents the orthogonal projection onto the range space of $\mathbf{HM}$, and $\mathbf{NN}^T$ represents the orthogonal projection onto the range space of $\mathbf{V}$. If the matrices $\mathbf{HM}$ and $\mathbf{V}$ are assumed full rank, the projection operators may be expressed in terms of the matrices $\mathbf{HM}$ and $\mathbf{V}$, respectively, and the expression for the measure ν is

$$\nu(\mathbf{V}, \mathbf{HM}) = \frac{\text{Trace}\, [\mathbf{V}(\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T\mathbf{HM}(\mathbf{M}^T\mathbf{H}^T\mathbf{HM})^{-1}\mathbf{M}^T\mathbf{H}^T]}{\alpha}. \tag{3}$$

As demonstrated in [14], a value of unity for ν characterizes a perfect set of filters. In (3), all matrices except the matrix $\mathbf{M}$ are known. The dimension of $\mathbf{M}$ is $N \times r$ where r is the number of scanning filters and N is the number of samples of a visible spectrum between 400–700 nm. In the examples discussed here, $N = 31$. Without any other restrictions, the measure ν is a function of rN parameters. The goal of filter design is to maximize the measure ν with respect to the rN parameters.

The measure ν is related to Neugebauer's q-factor by

$$\nu(\mathbf{V}, \mathbf{M}_H) = \frac{\sum_{i=1}^{\beta} q(\mathbf{o}_i)}{\alpha}$$

where $q(\cdot)$ is the q-factor with respect to the range space of $\mathbf{V}$, $\{\mathbf{o}_i\}_{i=1}^{\beta}$ is a set of orthogonal filters spanning the range space of the matrix $\mathbf{M}_H$, and α is the rank of $\mathbf{V}$.

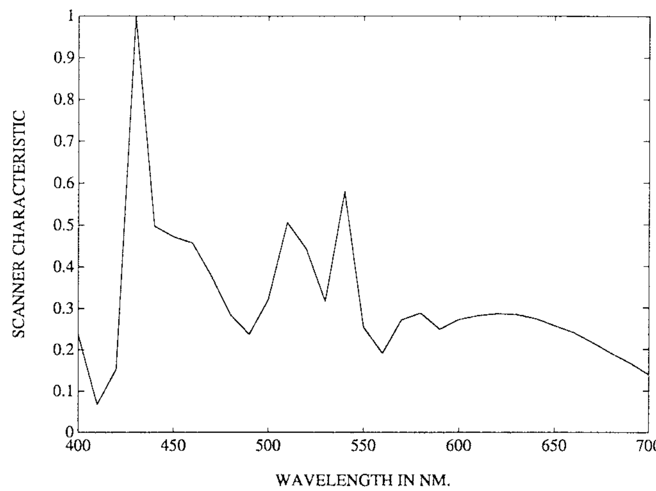

Fig. 1. Illuminant 1 or scanner characteristic 1. The combined effect of a particular scanning illuminant, optical path, and detector sensitivity. This represents the diagonal values of a particular matrix $\mathbf{H}$ used for illustration and experimental purposes.

B. The Problem of Optimal Filter Design

A simple and straightforward solution to the optimization problem is

$$\mathbf{m}_i(k) = \kappa_i \frac{\mathbf{v}_i(k)}{\mathbf{H}_{kk}} \qquad i = 1, 2, 3, \cdots s; \quad \mathbf{H}_{kk} \neq 0 \tag{4}$$

where $\mathbf{v}_i(k)$ is the kth element of the vector $\mathbf{v}_i$ and

$$\kappa_i = \left\{ \max_k \left[\frac{\mathbf{v}_i(k)}{\mathbf{H}_{kk}} \right] \right\}^{-1}$$

is the normalization constant for the ith filter, so that the maximum transmissivity of each designed scanning filter is unity. The set of the optimal filters of (4) consists of r filters such that $\mathbf{M}_H = \mathbf{V}\mathbf{\Lambda}_\kappa$ where $\mathbf{\Lambda}_\kappa$ is a diagonal matrix with diagonal values κ_i. The scanning system will replicate the vectors $\mathbf{v}_i$ exactly.

Consider the actual scanner characteristic $\mathbf{H}$ shown in Fig. 1 and assume $\mathbf{V} = \mathbf{A}$. The three scanning filters defined by (4) are shown in Fig. 2. Notice that the scanner characteristic is far from uniform, and that this leads to problems in filter design. The nonsmoothness of the scanner characteristic is a characteristic of the illuminant. This characteristic is not affected by the sampling rate. Smoothness of filter transmissivity curves is an important restriction in the filter fabrication process, and Fig. 2 indicates that filters designed according to (4) will not be easy to fabricate exactly. This makes it clear that each *constructable* filter does not possess N degrees of freedom and that expressing the measure as a function of rN independent variables will not necessarily result in optimal *realizable* filters.

III. Physically Realizable Filters

The measure defined above can be used to select from among a set of filters or as an optimization function with filter transmissivities as parameters for custom-designed filters.

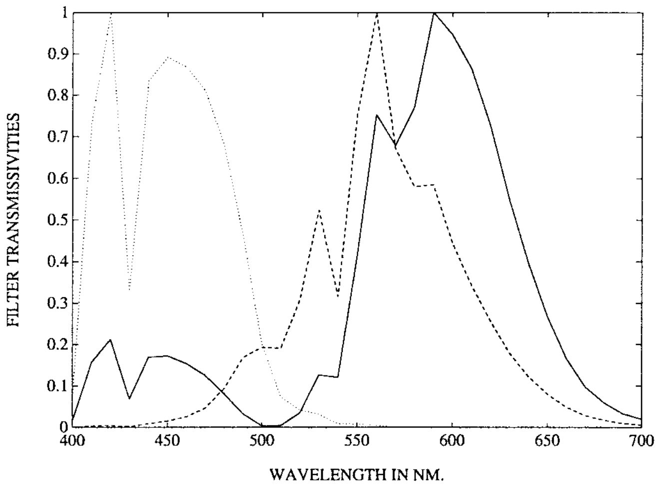

Fig. 2. Filters for the replication of the CIE matching functions with illuminant 1. Filters which, when installed in a scanner with characteristic represented by Illuminant 1, will replicate the CIE matching functions. The highly nonsmooth transmittance curves make these filters difficult to fabricate.

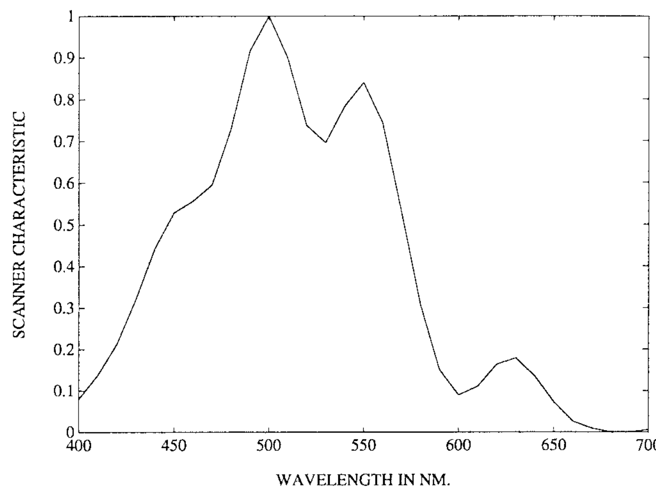

Fig. 3. Illuminant or scanner characteristic 2. The combined effect of another particular scanning illuminant, optical path, and detector sensitivity. This represents the diagonal values of a particular matrix $\mathbf{H}$ used for illustration and experimental purposes.

A. An Optimal Subset of a Discrete Set of Filters

A simple formulation of the optimization problem is to determine the "best" set of r filters from a set of existing filters. Suppose the set S is the set of existing filters from which the best subset M_0 of r filters is to be chosen. Expression (3) for the measure may be optimized with respect to subsets of S, of size r, by an exhaustive search taking the filters r at a time, each filter representing a scanning filter. If n is the size of set S, such a search will involve ${}^nC_r = n!/r!(n-r)!$ evaluations of the measure, where ${}^nC_r$ represents the number of subsets of size r of a universal set of size n. Heuristic methods that use specialized knowledge of the filter set, the scanning problem, or the particular data set could reduce the computational complexity of the search algorithm. The results of this approach have been reported in [18] and are summarized in Section IV.

B. Parametrization of Filter Characteristics

One way of incorporating a manageable dynamic range and smoothness for filters is by modeling each filter in terms of smooth, nonnegative functions of a few parameters. This section discusses the modeling of the filters as single Gaussians and as the sum of two Gaussians to illustrate the general procedure of parametrized optimization for scanning filter design. Other functions, such as raised-cosines, sums of raised-cosines, and exponential raised-cosines have been used [16]. The results obtained are not substantially different from those obtained using the Gaussian functions, and are hence not presented here. The total number of parameters is less than $5r$ in each case, resulting in tractable formulations of the optimization problem and in physically realizable filters. The functions were chosen for ease of implementation and efficiency of the optimization routine.

The functional form of the measure ν in terms of the parameters is not simple, and it would be very difficult to find a closed-form solution to the resulting optimization problem. It may be possible with current symbolic math software. Various existing optimization algorithms may be used to find points of local extrema of the measure ν with respect to the parameters. It is not, in general, possible to find global extrema for functions such as the measure ν. A common solution is to obtain a number of local extrema and take the best one among these as the estimate. In general, the local extremum obtained depends on the initialization of the optimization algorithm.

If each filter $\mathbf{m}_i$ is modeled as a weighted sum of Gaussian functions of means μ_{i1} and μ_{i2}, standard deviations σ_{i1} and σ_{i2} and weighting factor α_i, the normalized filter vectors $\mathbf{m}_i$ are

$$\mathbf{m}_i(k) = \exp\left[-\frac{(\lambda_k - \mu_{i1})^2}{2\sigma_{i1}^2}\right] + \alpha_i \exp\left[-\frac{(\lambda_k - \mu_{i2})^2}{2\sigma_{i2}^2}\right] \quad (5)$$

where λ_k depends on the sampling of the spectra. Each filter is a function of five independent variables, and the measure ν is a function of $5r$ independent variables. The resulting "optimal" filters will be sums of Gaussians and, hence, easy to fabricate. The number of parameters may be reduced to $2r$ by considering $\alpha_i = 0$, i.e., single Gaussian functions. While the smaller number of parameters makes finding the optimal values easier, it is shown in Section IV that this limitation results in poor performance. The results for the Gaussian filter models and the scanner characteristic in Fig. 1 and another actual characteristic shown in Fig. 3 are presented in Section IV.

C. Trimming Optimal Results

Often, it is desirable to obtain filters that are optimal with respect to a perceptual error measure such as the ΔE_{ab} error. Once a set of "optimal" (with respect to the measure ν) filters are found, it is feasible to "trim" these filters using a perceptual error. The average square ΔE_{ab} error

$$E_{sq} = \frac{\sum_{\mathbf{f}} \Delta E_{ab}^2(\mathbf{f})}{n} = \frac{\sum_{\mathbf{f}} [(L^* - L_f^*)^2 + (a^* - a_f^*)^2 + (b^* - b_f^*)^2]}{n} \quad (6)$$

is preferable to E_{ave} (2) as a criterion for trimming because it is easier to manipulate mathematically as mentioned in Section I. Note that E_{ave} is used for comparison in the tables.

Trimming involves finding a local minimum of E_{sq} over a particular data set, using optimal filters with respect to measure ν as the starting point for a steepest-descent algorithm [8, p. 285]. During trimming, it is not necessary to use parametrized models, and the filter transmittances at each wavelength are free variables. In the terminology of this paper, trimming is optimization of E_{sq} over a particular data set with respect to Nr parameters. Allowing the filter transmittances at each wavelength to be free variables is not expected to affect smoothness and nonnegativity of the filters greatly because the trimmed filters ought to be close to the optimal parametrized filters, which are smooth and nonnegative.

A mathematical expression for the average square E_{sq} error over a particular data set was obtained. This scalar expression was differentiated with respect to each of the rN variables. The gradient was used to approach a local minimum. Details of gradient calculation, as well as the final expression, can be found in [16].

IV. Experimental Results

The measure ν is first used to choose the best set of three commercial filters. While this solution is straightforward, it is also extremely practical and economical. The measure is then used to design custom interference filters for actual scanners. The optimal results thus obtained are trimmed as described in Section III-C.

A. A Subset of a Discrete Set

The Kodak Wratten filters, whose transmissivities are published, form the basic collection. The size of the Wratten filter set is approximately 100. This implies that the number of sets involved in finding the best set of three filters is $100!/3!97!$ or approximately 1.6×10^5. Results for an exhaustive search of the Wrattens to obtain the "best" set of three filters for $\mathbf{V} = \mathbf{A}, \mathbf{L} = \mathbf{I}$ and the scanner characteristic of Fig. 3 are tabulated. Table I shows the Wratten filter numbers for the optimal set and two other "good" sets, the value of the measure, $\nu(\mathbf{A}, \mathbf{HM})$, and the average ΔE_{Lab} error of the corrected measurements [14] over a 64 data-point subset of the set of standard Munsell color chips [22]. E_{ave} is the predicted value of expression (2) for the Munsell chip set, using tristimulus values calculated using knowledge of the spectra of the data set. The white point used for the data set was the white Munsell sample. Filter set 1 was installed in a scanner with the characteristic shown in Fig. 3 and used to scan the Munsell chip set. The resulting actual measured average ΔE_{ab} error, calculated from (2) using tristimulus values estimated from actual scanner measurements, was 3.02. The difference from the predicted value of 2.04 may be attributed to errors in the estimated scanner characteristic. Even with this difference, the results are a large improvement compared to an E_{ave} value of 4.5 obtained with the previously installed set designed by using the standard q-factor. These results have been reported in [18] and are included here for completeness.

TABLE I
Optimal Subsets of the Wratten Filter Set

Filter Set	Wratten Filter Nos.	Measure $\nu(\mathbf{A}, \mathbf{M}_H)$	E_{ave}
1	23A, 48A, 52	0.9122	2.04
2	9, 23A, 48A	0.9114	1.91
3	9, 48A, 52	0.9028	2.11

B. Parametrized Filter Models

For the particular scanner characteristics of Figs. 1 and 3, the parametrizations suggested in Section III-B were implemented to obtain the "best" set of three color scanning filters. The viewing illuminant was assumed uniform, i.e., $\mathbf{L} = \mathbf{I}$. To test the performance of the filters, the scanning process was simulated using the set of Munsell chips.

For all parametrizations, the MATLAB [25] function "fmins" was used to find optimal filters. This function is an implementation of the Nelder–Meade simplex algorithm. Clearly, the measure does not have one global maximum as, for example, the filters in a different order will give a different point in parameter space but the same value of the measure. To minimize this effect, several different initial points were used. The resulting filters gave varying but similar results (within 5%). Occasionally, the function returned a value that was clearly not optimal, e.g., $\nu < 0.6$, which was easily recognized and discarded.

The parameters for the single-Gaussian model were calculated for the scanner characteristics shown in Figs. 1 and 3. Table II shows the measure ν of the resulting optimal set of filters, the parameters defining each filter, and the average and maximum predicted ΔE_{Lab} errors E_{ave} and $E_{\max}$ over the subset of Munsell chips.

Barr Associates, a filter manufacturer, provided an estimate of the closest interference filters they could manufacture given the specifications for filter set 1 of Table II. Note that these filters were not manufactured and, hence, all errors reported here are estimated. This estimated realizable filter set had a measure ν of 0.9478, a value E_{ave} of 0.86, and a value $E_{\max}$ of 2.5. The sensitivity of the average square ΔE_{ab} error to filter fabrication errors is discussed in [17].

The parameter values obtained for the sum-of-Gaussian filter model of (5) are tabulated in Table III. The designed filters of set 2 are plotted as solid lines and the estimated filters (from Barr) are plotted as dotted lines in Figs. 4–6. The measure ν of the set estimated is 0.9900, E_{ave} is 0.50

TABLE II
1 PARAMETERS FOR SINGLE-GAUSSIAN MODEL

Filter Set	Illuminant	ν	Filter No. 1	Filter No. 2	Filter No. 3	E_{ave}	E_{max}
1	1	0.9485	$\mu_1 = 459.1$ $\sigma_1 = 22.0$	$\mu_2 = 557.7$ $\sigma_1 = 40.3$	$\mu_3 = 592.9$ $\sigma_1 = 35.8$	0.84	2.46
2	2	0.9556	$\mu_1 = 438.3$ $\sigma_1 = 26.4$	$\mu_2 = 548.0$ $\sigma_2 = 37.6$	$\mu_3 = 607.0$ $\sigma_3 = 19.1$	1.75	9.16

TABLE III
PARAMETERS FOR DOUBLE-GAUSSIAN FILTER MODEL

Filter Set	Illuminant	ν	Filter No. 1	Filter No. 2	Filter No. 3	E_{ave}	E_{max}
1	1	0.9698	$\mu_{11} = 461.6$ $\sigma_{11} = 23.4$ $\mu_{12} = 443.7$ $\sigma_{12} = 4.1$ $\alpha_1 = 0.4558$	$\mu_{21} = 559.7$ $\sigma_{21} = 7.6$ $\mu_{22} = 549.3$ $\sigma_{22} = 43.8$ $\alpha_2 = 0.9552$	$\mu_{31} = 585.2$ $\sigma_{31} = 27.6$ $\mu_{32} = 622.0$ $\sigma_{32} = 25.5$ $\alpha_3 = 0.5897$	0.75	3.28
2	2	0.9928	$\mu_{11} = 442.7$ $\sigma_{11} = 24.6$ $\mu_{12} = 428.2$ $\sigma_{12} = 7.0$ $\alpha_1 = 0.3022$	$\mu_{21} = 593.6$ $\sigma_{21} = 14.7$ $\mu_{22} = 539.9$ $\sigma_{22} = 29.0$ $\alpha_2 = 0.5281$	$\mu_{31} = 601.2$ $\sigma_{31} = 11.2$ $\mu_{32} = 638.1$ $\sigma_{32} = 49.4$ $\alpha_3 = 0.3969$	0.46	1.71

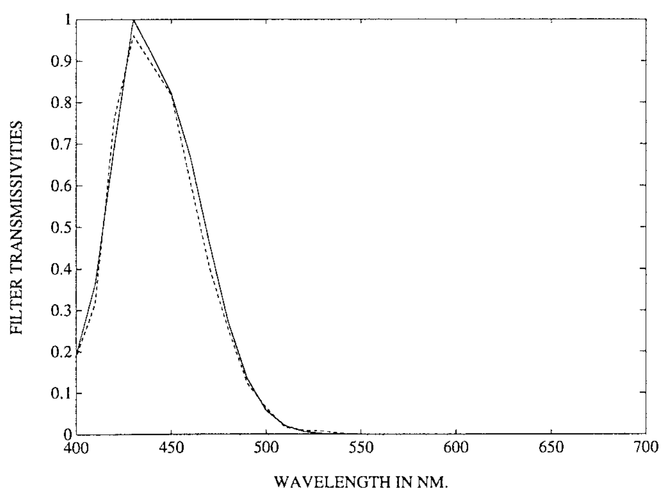

Fig. 4. Designed and estimated (blue) filter for double-Gaussian model and illuminant 2. Solid: Designed filters. Dashed: Estimated filters.

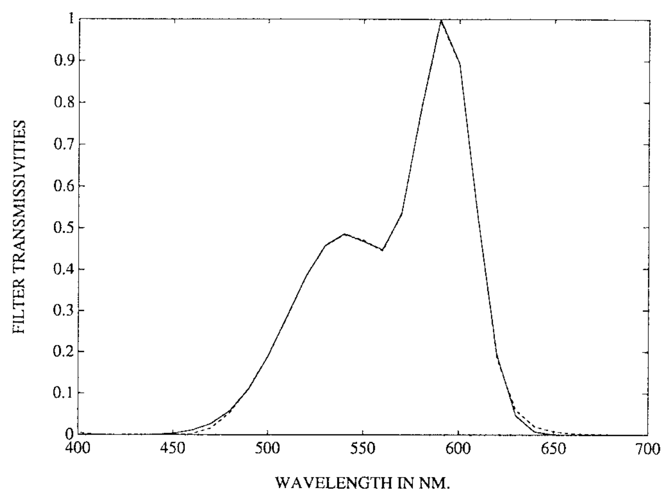

Fig. 5. Designed and estimated (green) filter for double-Gaussian model and illuminant 2. Solid: Designed filters. Dashed: Estimated filters.

and $E_{\max}$ is 1.95. This indicates that the design method can produce excellent practical results. The effective scanning filters of set 1, i.e., the combined effect of the designed Filter set 1 and Illuminant 1, are plotted in Fig. 7. These plots illustrate that the effective filter set need not be "close" to the CIE matching functions for a "good" filter set.

Results of the method for a set of thin-film filters that were actually fabricated are presented in [19]. The filters were designed for the scanner with illuminant 2. The predicted E_{ave} was 1.25 for the designed set, 1.99 for the fabricated set. The measured E_{ave} was 2.66. The result was higher than the prediction but still a significant improvement over the best Wratten set of Section IV-A.

In general, an increase in the number of parameters used to define the filters should give better results. Experiments were performed to obtain the best sum-of-three-Gaussians for the two illuminants. Using the optimal results for sum-of-two-Gaussians presented in the previous section as initial estimates did not result in substantially different filters. Other initial points provided slightly higher values. Representative results for one final point for each scanner characteristic are presented in Table IV. Note that the values of the measure are larger than the corresponding values of the measure listed in Table III for the sum-of-Gaussian model for either illuminant, and that the values of E_{ave} and $E_{\max}$ are considerably smaller for illuminant 2. The fact that the ΔE_{ab} errors are below vision

TABLE IV
PARAMETERS FOR SUM-OF-THREE-GAUSSIAN FILTER MODEL

Filter Set	Illuminant	ν	Filter No. 1	Filter No. 2	Filter No. 3	E_{ave}	E_{max}
1	1	0.9722	$\mu_{11} = -144.1$	$\mu_{21} = 575.5$	$\mu_{31} = 593.6$	0.73	3.0
			$\sigma_{11} = 33.0$	$\sigma_{21} = 43.8$	$\sigma_{31} = 17.1$		
			$\mu_{12} = 441.1$	$\mu_{22} = 624.8$	$\mu_{32} = 554.8$		
			$\sigma_{12} = 8.9$	$\sigma_{22} = 15.6$	$\sigma_{32} = 10.6$		
			$\mu_{13} = 466.2$	$\mu_{23} = 555.1$	$\mu_{33} = 608.3$		
			$\sigma_{13} = 18.4$	$\sigma_{23} = 3.6$	$\sigma_{33} = 38.5$		
			$\alpha_{12} = 0.8158$	$\alpha_{22} = -0.2314$	$\alpha_{32} = 0.8429$		
			$\alpha_{13} = 1.4860$	$\alpha_{23} = 1.3712$	$\alpha_{33} = 0.8846$		
2	2	0.9951	$\mu_{11} = 430.4$	$\mu_{21} = 592.9$	$\mu_{31} = 626.7$	0.23	1.19
			$\sigma_{11} = 13.2$	$\sigma_{21} = 16.3$	$\sigma_{31} = 41.9$		
			$\mu_{12} = 450.7$	$\mu_{22} = 538.3$	$\mu_{32} = 601.3$		
			$\sigma_{12} = 36.2$	$\sigma_{22} = 27.8$	$\sigma_{32} = 10.9$		
			$\mu_{13} = 460.2$	$\mu_{23} = 594.3$	$\mu_{33} = 674.1$		
			$\sigma_{13} = 12.6$	$\sigma_{23} = 4.7$	$\sigma_{33} = 11.8$		
			$\alpha_{12} = 0.1898$	$\alpha_{22} = 0.5832$	$\alpha_{32} = 0.2570$		
			$\alpha_{13} = 0.5656$	$\alpha_{23} = 0.4971$	$\alpha_{33} = 0.2371$		

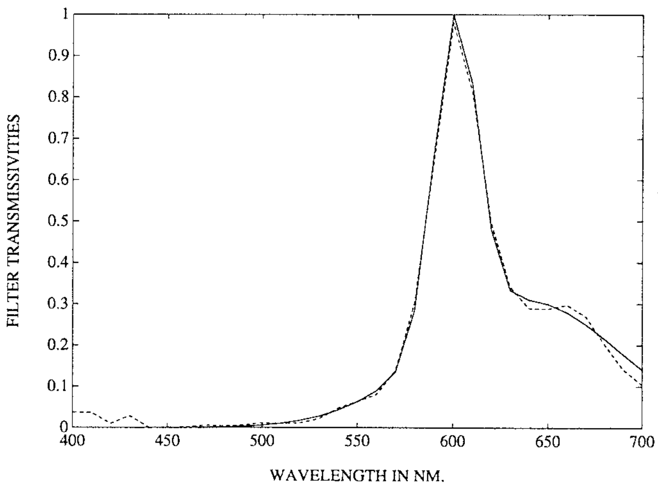

Fig. 6. Designed and estimated (red) filter for double-Gaussian model and illuminant 2. Solid: Designed filters. Dashed: Estimated filters.

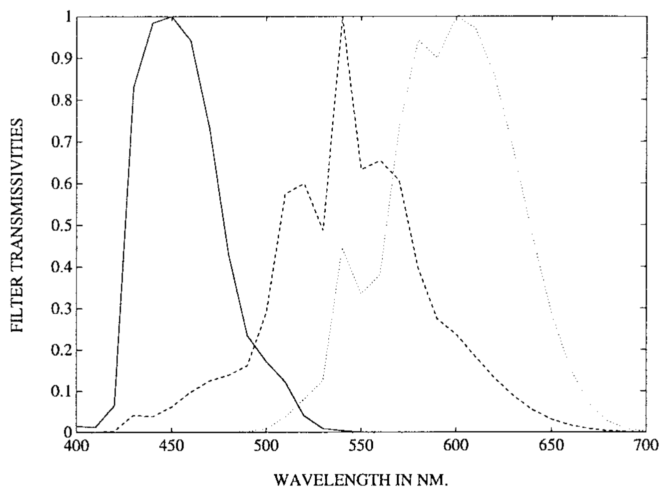

Fig. 7. Designed effective scanning filters for double-Gaussian model and illuminant 1. Combined effect of scanning filters and scanner characteristic. If the filters replicated the CIE matching functions, as those in Fig. 2, the effective scanning filters would be the CIE matching functions themselves. It is clear from these that good filters need not be close to the CIE matching functions.

perception [11] implies that it may not be necessary to increase the number of parameters further. Furthermore, the sum of a larger number of Gaussians could result in a multimodal curve, which might be difficult to fabricate.

C. Filter Trimming

The gradient of E_{sq} (6) over the Munsell chip set was used to trim the single-Gaussian designs for illuminants 1 and 2 and the double-Gaussian for illuminant 2 [16]. The trimming was performed using steepest descent programmed in MATLAB using the mathematical expression for the gradient derived in [16]. The step size for the steepest descent was chosen heuristically, between 0.1 and 0.001, and decreased as the algorithm progressed. The iterative procedure was terminated when small-valued peaks started appearing in the filter transmittance curves, at wavelengths where filter transmittance values for the starting point of the gradient descent were close to zero.

The results of trimming the single-Gaussian filters are shown in Figs. 8–10. Note that the filter shape has changed sufficiently so that the trimmed filters may no longer be modeled as single Gaussians, though the trimmed filters retain the smoothness of the original designs.

Table V lists the different error measures before and after trimming. The root mean square ΔE_{ab} error over the Munsell chip set is denoted RMS. It is not identical to E_{ave}. The value of RMS is indicative of the improvement in E_{sq}. It is clear that trimming improves the general performance of the filter set. The single-Gaussian filter sets have shown more improvement than the sum-of-Gaussian set because the sum-of-Gaussian set was a better set initially. Note that the measure ν may decrease slightly due to trimming, since the trimming is based on decreasing the data-dependent ΔE_{ab} error. While increasing

TABLE V
COMPARISON BETWEEN ERRORS BEFORE AND AFTER FILTER-TRIMMING

Filter Model	Before Trimming				After Trimming			
	ν	E_{ave}	E_{max}	RMS	ν	E_{ave}	E_{max}	RMS
Single-Gaussian (Illuminant 1)	0.9485	0.84	2.46	1.09	0.9508	0.27	0.67	0.32
Single-Gaussian (Illuminant 2)	0.9556	1.75	9.16	2.49	0.9417	0.41	2.18	0.58
Double-Gaussian	0.9928	0.46	1.71	0.60	0.9918	0.12	0.73	0.18

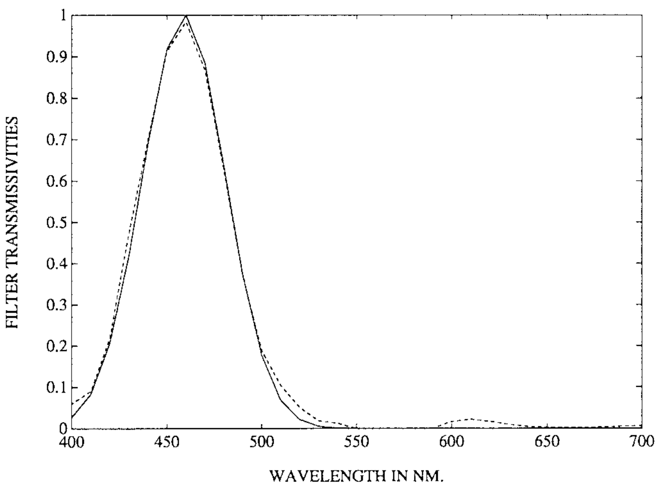

Fig. 8. Trimmed blue filter for single-Gaussian model and illuminant 1. Solid: Designed filters. Dashed: Trimmed filters.

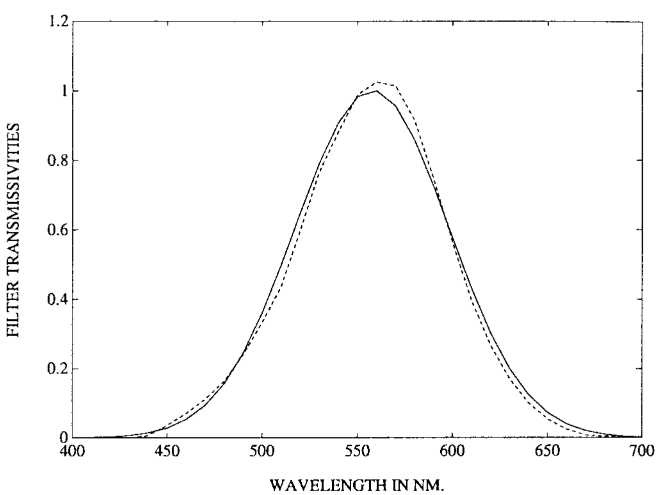

Fig. 9. Trimmed green filter for single-Gaussian model and illuminant 1. Solid: Designed filters. Dashed: Trimmed filters.

ν is highly correlated to decreasing ΔE_{ab}, the relation is not deterministic. Trimming is data dependent because it uses the data-dependent E_{sq} as an optimization criterion. This provides a closer-to-optimal solution with respect to perceptual error, for the particular data set.

V. CONCLUSIONS

The measure of goodness of a set of color filters [14] may be used to define an optimization criterion. The direct application is used to choose the best set from a collection of filters and hardware implementation indicates that this method is very useful for choosing filters for colorimetric applications. The modeling of the filters results in a parametrization of the filter design problem. This problem may be satisfactorily solved by standard minimization (or maximization) routines to give filters with fairly high measures ν. It was shown that solutions for real scanner characteristics could be fabricated as interference filters with negligible performance degradation. The gradient of the average square ΔE_{ab} error may be used to trim the optimal solutions to significantly improve the parametrized solutions to produce smooth filters with low ΔE_{ab} errors.

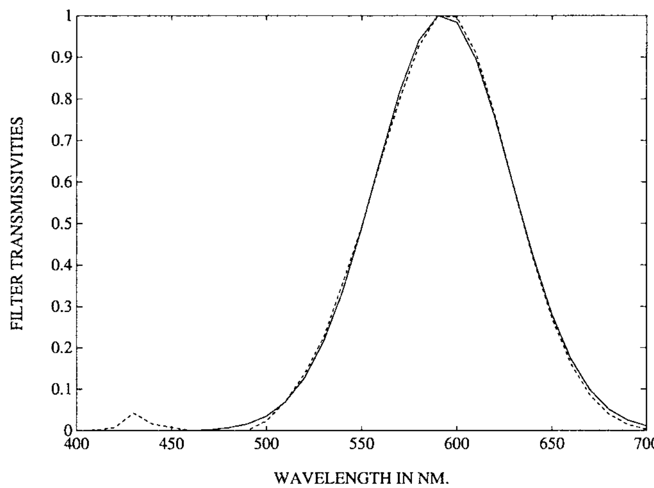

Fig. 10. Trimmed red filter for single-Gaussian model and illuminant 1. Solid: Designed filters. Dashed: Trimmed filters.

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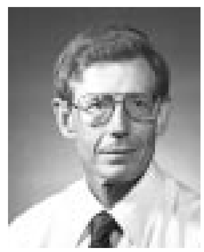

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